Optimal Resilience in Multi-Tier Supply Chains*

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Abstract

Forward-looking investments determine the resilience of firms' supply chains. Such investments confer externalities on other firms in the production network. We compare the equilibrium and optimal allocations in a general equilibrium model with an arbitrary number of vertical production tiers. Our model features endogenous investments in protective capabities, endogenous formation of supply links, and sequential bargaining over quantities and payments between firms in successive tiers. We derive policies that implement the first-best allocation, allowing for subsidies to input purchases, network formation, and investments in resilience. The first-best policies depend only on production function parameters of the pertinent tier. When subsidies to transactions are infeasible, the second-best subsidies for resilience depend on production function parameters throughout the network, and subsidies are larger upstream than downstream whenever the bargaining weights of buyers are non-increasing along the chain.

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1 Introduction

A spate of highly publicized supply chain disruptions—owing not only to the COVID-19 pandemic, but also to natural disasters, cyber-attacks, extreme weather events, logistics bottlenecks, geopolitical tensions, and a host of other causes—have drawn policymakers' attention to the importance of supply chain resilience. International institutions such as the O.E.C.D (2021) and European Parliament (2021) have issued reports with "resilience" or "robustness" in their titles.[1] Government publications, such as the U.K. Department of International Trade (2022) and the U.S. *Economic Report of the President* (Council of Economic Advisors, 2022, chapter 6), and international organizations such as the World Bank (2023), have also addressed these issues. Think tanks, such as McKinsey Global Institute (Lund et al., 2020) and the Brookings Institution (Iakovou and White, 2020), have offered guidance as well. Yet little formal economic analysis has addressed the topic of optimal government policy in the face of ongoing risk of supply chain disturbances.

In this paper, we examine the market failures that may generate sub-optimal resilience in complex supply chains. We seek to capture in a stylized but realistic way one of the canonical supply-chain forms described in Lund et al. (2020) and the *Economic Report of the President* (Council of Economic Advisors, 2022); see Panel B of Figure 6.1 in the latter.[2] In what that report calls "outsourcing with isolated industries," inputs travel downstream through several or many tiers until they are ultimately transformed into a consumer good. Lead firms create the product designs and oversee specifications, at least from their immediate suppliers if not further up the chain, but they typically do not own or control most of these suppliers. Often, sourcing takes place *sequentially* (Yoo et al., 2021) and lead firms (a.k.a. original equipment manufacturers, or OEMs) *delegate procurement* of components to their upstream partners (Guo et al., 2010). These features of sequential and delegated procurement are described more fully in Mena et al. (2013) and the references therein.

The McKinsey report describes another salient characteristic of modern supply chains, namely the large numbers of firms that are typically involved. They examined lists of publicly-disclosed suppliers for 668 large manufacturing companies and report that most have hundreds of direct suppliers, who collectively have thousands of suppliers in the tier above. For example, General Motors reports 856 direct suppliers and a total of more than 18,000 suppliers to those direct suppliers. For Apple, those numbers are 638 and more than 7,400, respectively, while for Nestlé they are 717 and more than 5,000. Moreover, Carvalho and Tahbaz-Salehi (2019) observe that

[1] Baldwin and Freeman (2022) cite the business literature to distinguish between resilience and robustness. They describe resilience as "the ability of organizations and supply chains to plan for, respond to, and recover from disruptions in a timely and cost-effective manner" (Martins de Sá et al., 2019) and robustness as "the ability to maintain operations during a crisis (Brandon-Jones et al., 2014). In our static framework, we cannot distinguish between these two concepts, and so we use the term resilience to refer to both of forms of protection from disruptions.

[2] Baldwin and Venables (2013) coined the terms "snake" and "spider" to distinguish supply chains in which an input passes through multiple stages with sequencing dictated by engineering considerations from chains that involve the assembly of parts in no particular order. They focus on the effects of a reduction in international frictions on the location of production in these alternative types of global supply chains. Our model is something of a hybrid, with a spider structure at every tier and a snake structure that links the different tiers.

input suppliers often sell to several or many lead firms. For example, Dell and Lenovo share 2,272 direct suppliers among the total of 7,033 serving the former company and the 6,240 serving the latter (Lund et al., 2020, p.9).

Guided by these observations, we develop a novel, general-equilibrium model of network production featuring multi-tier supply chains, arms-length transactions between firms in different layers, many input suppliers for each manufacturer and many customers for each intermediate producer, and sequential procurement. The supply chains that we envision do not involve off-the-shelf inputs that might be available on anonymous markets. Rather, they are produced and sold to order. In our model, each producer negotiates the terms of its purchase contract with each of its potential suppliers. The contracts specify the quantities that will be delivered by the upstream firms and the payments that will be made in return. Transactions take place only between firms that have borne the prior fixed costs of forming relationships. In this setting, we introduce risks of disruption at every node along the chains.

More specifically, we model an economy with a finite measure of firms that produce differentiated consumer goods and sell them to households in a setting of monopolistic competition. These lead firms, which are active in what we denote by tier S, produce their unique varieties using labor and bundles of differentiated intermediate inputs that they purchase from firms operating in tier $S-1$. The firms in tier $S-1$, in turn, fulfill their orders by combining labor and differentiated inputs procured from their partners in tier $S-2$. Firms in tier $S-2$ buy inputs from suppliers further upstream, and so on up the chain. The vertical chain ends with tier 0, where companies produce inputs from labor alone and sell them to firms in tier 1.

Since each supplier has many customers and each customer has many suppliers, and since firms have overlapping but different networks, it would be impractical for a grand negotiation to take place among all firms in the economy. Instead, we assume cooperative but simultaneous bargaining among isolated pairs in adjacent tiers. We assume a Nash-in-Nash equilibrium for the bargaining outcomes between all firms in some tier s and those in tier $s-1$ (Horn and Wolinsky, 1988); that is, each member of a pair takes as given the outcomes of its negotiations with all of its other suppliers or buyers, as the case may be. Meanwhile, we impose a sequential structure to the series of negotiations across tiers, in keeping with a prominent strategy described by Yoo et al. (2021).[3] Bargaining begins with negotiations between firms in tier S and their suppliers in tier $S-1$ and proceeds upstream until firms in tier 1 sign contracts with firms in tier 0. All pairs are forward looking, recognizing that their agreements have implications for their subsequent purchases and payments both on and off the equilibrium path.

We assume that every firm faces a positive probability of a catastrophic supply disruption. If a firm suffers such a disturbance, it will be unable to produce in the period captured by the model. The risks of disruption depend on actions undertaken by the firms to foster resilience and may vary across tiers of the supply chain. A firm's profits depend on its own fate and that of all of its

[3] Yoo et al. (2021) cite the example of Google, which outsources the manufacturing of its built-in streaming technology Chromecast to Flex, while delegating to Flex the sourcing decisions from second-tier suppliers.

suppliers and customers.

To capture the private opportunities available to promote supply chain resilience, we grant firms two means to moderate their risks. First, firms may invest in *protective capability*, which MacDuffie et al. (2021, p.20) define as "the ability of firms to minimize damage inside facilities, sites and routes of the supply chain." Firms might choose to install equipment and erect buildings that are protected from weather shocks, establish strict health and safety protocols, design facilities that inhibit the spread of disease, and invest in cybersecurity. Under the heading of protective capability, we would also include what *The Economic Report of the President* (2022, p. 212) refers to as investments in *agility*, by which they mean "workers' ability to solve problems that ... enabl[es] them to pivot quickly to alternative products or processes or react to abnormal situations." In short, we allow firms to devote resources to reducing the probability that their own operations will be disrupted.

Second, we allow firms to invest in *network thickness*. Each firm chooses the fraction of suppliers in the tier immediately above its own with whom it forms relationships. Having multiple suppliers protects a firm against the event that some of its partners are unable to produce. *The Economic Report of the President* (2022, p. 211) describes a thick network as providing *redundancy*, that is, the wherewithal to replace a particular input supplier with another that offers a close substitute. In our model, where firms demand a variety of inputs, none of which is critical to its operation, a thicker network directly boosts productivity in the face of supplier outages. We assume that developing relationships is costly, as potential suppliers must be identified, vetted, instructed about specifications, and have their prototypes tested for quality.

Our analysis focuses on the "wedges" that emerge between private and social incentives at different stages of the supply chain. To identify these wedges, we solve a planner's direct-control problem and then ask what instruments the government would need to implement the first-best allocation as a decentralized equilibrium. We do not interpret these "optimal policies" literally as a prescription for industrial policy. Rather, the optimal policies help us to identify where inefficiencies can arise in arms-length supply chains, how the extent of these inefficiencies might vary across tiers that differ in their place in the chain, and how the inefficiencies in a given tier reflect conditions in other parts of its network.

In general, the government would need three types of policy instruments in our setting to achieve the first best: a set of subsidies or taxes on transactions between firms in adjacent tiers; a set of subsidies or taxes to promote or discourage investments in protective capability in different tiers; and a set of subsidies or taxes to encourage or impede the formation of supplier relationships. The first-best transaction subsidy for any pair of firms depends only on the bargain weights and production parameters for that dyad. The optimal policies to promote first-best resilience depend only on the bargaining weight that a firm achieves in its negotiations with its customers and on the size of the optimal subsidy for its sales to those customers.

We find that the outcome of each bargaining game yields an intuitive "markup factor" relating the price paid for inputs by firms in some tier to the production cost for the firms in the tier above. The endogenous markup reflects the relative bargaining weights of the upstream and downstream firms and the substitutability between the various inputs used by the latter. The optimal transaction subsidy counteracts the effect of the markup on marginal cost, much as in settings with imperfectly-competitive *markets* (rather than bilateral bargaining) for *standardized* inputs.

The optimal policy to promote or discourage investments in protective capability reflects two offsetting considerations. On the one hand, such investments confer a positive externality to the clients immediately downstream in a firm's network. On the other hand, the subsidy to transactions that is part of the first-best policy package inflates the private profitability of investments in resilience relative to their social value. If bargaining and technology parameters are common across tiers, then the first-best subsidies to resilience do not vary with a good's place in the supply chains, except for those at the extreme ends of the chain.[4] Alternatively, if goods further downstream are more differentiated than those upstream and other production and bargaining parameters are the same, the optimal subsidies for resilience decline as a good proceeds downstream. In any case, the optimal "subsidy" for investments in protective capabilities by firms in any middle tier may in fact be a *tax*, if the first-best subsidy for input purchases by those firms is large enough. Finally, we show that the optimal subsidies for network formation are the same as those for protective capability, despite the fact that firms have a private incentive to use these investments to improve their bargaining position vis-à-vis their suppliers and buyers.

It is perhaps surprising that the first-best policies do not depend on parameters that describe a firm's entire production network. After all, when a firm becomes better protected against supply disruptions or creates a larger network, the greater productivity that results from its presence or from its greater number of suppliers confers a *positive* externality to other companies upstream and downstream in the firm's own network, while conferring a *negative* externality on firms in other networks, including those in its own tier. We show, however, that in the presence of optimal subsidies to counteract the distorting effects of the negotiated markups, these positive and negative spillovers to firms that are not direct suppliers cancel in the general equilibrium. What remain are only the benefits that accrue to the firm's immediate customers and the wedge between social and private returns to investment that results from the transaction subsidies.

As we have noted, the first-best policies for investments in protective capabilities and network formation reflect the fact that the government uses subsidies for input purchases to ensure the ideal sizes of tier-to-tier transactions. But such subsidies may be politically sensitive, if they are viewed as handouts to the corporate sector. Given the public focus on resilience, we feel it is interesting also to examine a second-best setting in which policies to promote protective capability and thicker networks are used in the absence of subsidies to transactions. We find that the second-best policies differ from the first-best policies not only in magnitude, but also in the information that enters into their design. Whereas the first-best subsidies to investments in resilience depend only on technological parameters relevant to the tier being targeted, the second-best policies reflect technological parameters that describe the entire supply chain. Specifically, the

[4] Some authors, like Antràs et al. (2012), refer to the place of an industry in the supply chain as the degree of its "upstreamness." Our finding says that, with common production parameters and bargaining weights in all tiers, the first-best subsidy for resilience is independent of this characteristic of an industry.

second-best subsidies reflect, among other considerations, an input's place in the supply chain.

Although our main focus here is on the policy imperative that arises from the risk of supply disturbances, our paper also contributes a new model to the toolkit on supply chains. Our model is distinctive in its combination of vertical chains with multiple tiers, endogenous network formation, endogenous investments in protective capabilities, bilateral and sequential bargaining, and general equilibrium. Models of endogenous networks such as Oberfield (2018), Acemoglu and Azar (2020) and Kopytov et al. (2022), typically assume roundabout production processes, whereas those with vertical chains such as Ostrovsky (2008), Antràs and Chor (2013) and Johnson and Moxnes (2023) often take the network as given. Like us, Dhyne et al. (2023) allows for costly investments in supplier relationships, but in their case the probability of supply failures is completely exogenous and downstream firms subsequently purchase inputs from their suppliers at marginal cost.

Many of the supply chains modeled in the literature are fully efficient, either because a lead firm organizes all the transactions along the chain (Antràs and de Gortari, 2020), because the market structure is perfectly competitive (Kopytov et al., 2022; Johnson and Moxnes, 2023), or because a stability mechanism weeds out inefficient pairings (Oberfield, 2018). These models are not suitable for studying the externalities that arise from investments in protective capability and network thickness, which are the main focus of our analysis.[5]

This paper shares some of the concerns addressed in Grossman et al. (2023), although the economic environments in the two papers are very different. Grossman et al. (2023) use a simple production structure in which a single critical input is used in fixed proportion to final output. Each final producer can purchase its sole input at marginal cost from any supplier with whom it has a prior relationship that survives a potential supply disruption. The focus of that paper is on whether firms have adequate incentive to diversify their sourcing across locations and whether they have appropriate incentive to source in a safer, high-cost country relative to a riskier, low-cost country. There are no investments available to reduce the risk of a disruption and no reasons for a firm to invest in a thicker network aside from providing insurance against the loss of its critical input. Here, we are primarily interested in how distortions differ upstream versus downstream, which demands a setting with multi-tier supply chains. We capture the empirical observation that firms in supply chains have many suppliers and customers, and we model explicitly the bargaining that determines quantities and payments. We also endogenize the probabilities of shocks by allowing firms to invest in protective capabilities. To handle this richer environment, we abstract from critical inputs and from shocks that are common to all firms in a given country.

Our paper also bears some similarity to recent, independent work by Acemoglu and Tahbaz-Salehi (2024). They too study supply chains with endogenous networks that result from costly relationship-specific investments. In their model, like ours, transactions reflect negotiations between isolated pairs of firms, although there are some important differences in the details of the bargaining protocols.[6] Their supply chains have neither a vertical nor a sequential structure, and they do

[5] Few models allow for negotiated prices and quantities along the chain. An exception is Alviarez et al. (2023), but they allow for only two production tiers and have no investments in resilience or network formation.

[6] Acemoglu and Tahbaz-Salehi (2024) assume that firms can negotiate contracts with two-part tariffs that are

not consider ongoing risks of supply disturbances. Instead, they focus on the macroeconomic propagation of a single, unanticipated shock and especially on how small shocks can generate large changes in aggregate output due to the endogenous dissolution of supply relationships. Although they comment on the inefficiency of equilibrium with endogenous networks, they do not consider the optimal policy response at different points along the supply chain.

Like us, Elliot et al. (2022) study supply chain disturbances with idiosyncratic risks of failure. In their decentralized equilibrium, firms source inputs from multiple suppliers and invest resources to strengthen their relationships. However, there are several differences between their setting and ours. In their model, each firm has a finite set of critical inputs (much as in Grossman et al., 2023). Also, the microfoundations that they provide in their Appendix feature roundabout production, not vertical relationships. Their formulation does not allow for bilateral bargaining to determine quantities and prices. Finally, they address the determinants of resilience only in a single supply chain, because the complexity of their model precludes a general-equilibrium analysis.

There is an interesting parallel between our findings concerning second-best policies to promote resilience and results reported in Liu (2021) on optimal "industrial policies." Liu introduces exogenous wedges into a generic model of production networks. When the networks have a vertical structure, as here, the government's second-best policy is to provide larger production subsidies to sectors that are relatively farther upstream.

Finally, it is worth emphasizing that, in this paper, we treat only networks that form in a closed economy. In contrast, Antràs and Chor (2013), Antràs and de Gortari (2020), Grossman et al. (2023), Alviarez et al. (2023), Johnson and Moxnes (2023) and Fontaine et al. (2023), among others, deal with issues of international specialization in *global* supply chains. We hope to study optimal policy in the open economy in our future research.

To reiterate, our main contribution in this paper is to provide a rich yet tractable framework that can be used to study complex investment decisions in supply chains. Our model features an arbitrary number of tiers, bilateral bargaining, costly supplier relationships, and investments in protective capability. It captures several realistic externalities that arise in this setting and we provide a complete characterization of first-best and second-best policies for a closed economy.

The remainder of our paper is organized as follows. In the next section, we develop our model and describe the outcomes of the sequential bargaining and the equilibrium choices of investments in resilience and network formation. In Section 3, we study the first-best allocation, outlining first the solution to the planner's direct-control problem and then the policies that a benevolent government can use to implement the optimum as a decentralized equilibrium. We characterize in turn the optimal subsidies for input transactions, for investments in resilience, and for the formation of supplier relationships. Section 4 addresses the second-best policy problem that arises when the

contingent on the realized production networks. In effect, all bilateral contracts are renegotiated when any negotiation breaks down. By allowing for renegotiation, they eliminate any inefficiencies in the sizes of equilibrium transactions between firms in an equilibrium network and focus instead on inefficiencies in the extensive margin of the equilibrium network. In contrast, our analysis admits "double marginalization" that affects both the sizes of transactions and the incentives for investments in supplier relationships and in protective capabilities. See Lee et al. (2021, sec 4.2) for a discussion of the empirical literature that established the importance of double marginalization in several industries.

government cannot subsidize transactions, but can only promote (or discourage) investments in resilience and network formation. Section 5 concludes.

2 A Model of Multi-Tier Supply Chains

In this section, we develop a general-equilibrium model of vertical supply chains with an arbitrary number $S+1$ of production tiers and risks of supply disruptions throughout. A firm in the uppermost tier 0 produces a differentiated intermediate input using labor alone. A firm in a middle tier $s \in \{1, 2, \ldots, S-1\}$ produces an intermediate using labor and a bundle of inputs from tier $s-1$. It procures this bundle by bargaining over quantities and payments with the various suppliers in its production network. A firm in tier S produces a differentiated consumer good using labor and a bundle of tier $S-1$ inputs. We take the measure of firms in each tier s as given, and denote this measure by N_s for $s \in \{0, 1, \ldots, S\}$.[7]

2.1 Overview and Notation

As a guide to what follows, we begin with a brief overview of the model and notation. We do so with reference to two figures that describe, respectively, the timing in the model and the transactions between successive tiers.

Figure 1 portrays the timing. First, firms invest in their protective capabilities and form links with potential suppliers. We let r_s denote the extent of the investments in things like weather-proofing and cybersecurity by firms in tier s. Such investments reduce the probability $1 - \phi_s(r_s)$ that the firm will suffer a catastrophic supply disruption, with $\phi_s'(r_s) > 0$ and $\phi_s''(r_s) < 0$ for all $s \in \{0, 1, \ldots, S\}$. Meanwhile, a typical firm in tier s, $s \in \{1, 2, \ldots, S\}$, elects to form relationships with the fraction η_s of the N_{s-1} suppliers in tier $s-1$ at a cost of k units of labor per relationship.

In the next stage, disruption shocks are realized that disable a fraction $1 - \phi_s$ of firms in tier s, leaving a measure $\phi_s N_s$ of active firms. In the main text, we shall assume that all surviving firms in a tier have the same productivity, which we normalize to equal one. But in the appendix, we develop a more general version of the model in which surviving firms draw a Hicks-neutral productivity parameter from a known probability distribution with density function $f_s(z)$, as in Melitz (2003). We show in the appendix that the policy conclusions for the model with heterogeneous firms are identical to those in the model with similar firms in a given tier.

Firms that survive the supply disturbances move on to the procurement stage. Procurement takes place sequentially. First, the lead producers negotiate with their surviving suppliers in tier $S-1$. These negotiations take place simultaneously and the negotiants take all other bargaining outcomes as given. After these end-of-chain bargaining has been concluded, firms in tier $S-1$ bargain simultaneously with suppliers in tier $S-2$. This sequential bargaining continues until finally firms in tier 1 sign contracts with firms in tier 0.

[7] We could readily allow for free entry at some fixed costs that vary by tier. This would not change any of our results regarding the first best, provided the government can also subsidize or tax entry.

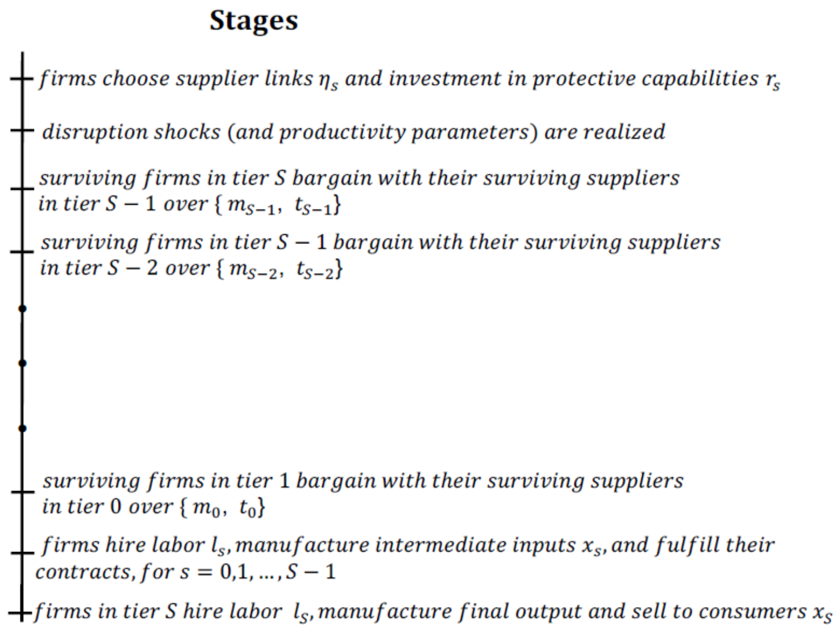

Figure 1: Sequence of Events and Decisions

Figure 2 depicts the sourcing in more detail. First notice that each buyer has multiple suppliers and that each supplier has multiple customers. For example, firm F in tier s supplies inputs to producers K, L and M in tier $s+1$, while procuring inputs from firms C and D in tier $s-1$. The network for firm F overlaps with that of firm G, but not perfectly so. A firm in tier s negotiates a contract with each of its suppliers in tier $s-1$ that calls for a quantity of inputs, m_{s-1}, and a payment of t_{s-1}.[8] In the extended model with heterogeneous firms outlined in the appendix, the quantities and payments are functions of the productivity of the buyer and the productivity of the supplier. In any case, the Nash bargaining gives weight β_s to the buyer in tier s and the weight $1-\beta_s$ to the supplier in tier $s-1$, as noted in the figure.[9]

After all the contracts have been negotiated, the firms in tier s hire l_s units of labor to combine with their input purchases of m_{s-1} units from each of their $n_s^u \equiv \eta_s \phi_{s-1}(r_{s-1}) N_{s-1}$ suppliers to produce x_s units of output. Again, if firms in tier s are heterogeneous in productivity—as outlined in the appendix—then l_s and x_s will be functions of the productivity of the producer, and m_{s-1} will be a function of both the productivity of the producer and that of the particular supplier. Finally, the lead producers in tier S hire l_S units of labor, produce x_S units of output, and sell

[8] Equivalently, the firms could negotiate a quantity and a per-unit price. As in other settings with cooperative bargaining, the firms set the quantity that is jointly optimal, then share the surplus by choice of payment. It follows that we could as well specify that firms negotiate two-part tariffs, as in Acemoglu and Tahbaz-Salehi (2023), with a fixed payment and a price per unit, and then they could allow the buyer to choose the quantity unilaterally.

[9] Although the figure depicts a setting with discrete numbers of suppliers and customers, this is for illustrative purposes only. The analysis below treats the case of a continuum of firms. We solve the bargaining problem with "the last firm" by differentiating benefits and costs with respect to the measure of firms and allowing the bargain at the margin to differ from those with the remaining firms. Each firm enjoys a small surplus from the marginal transaction and the Nash bargaining solution applies to these small surpluses, as usual.

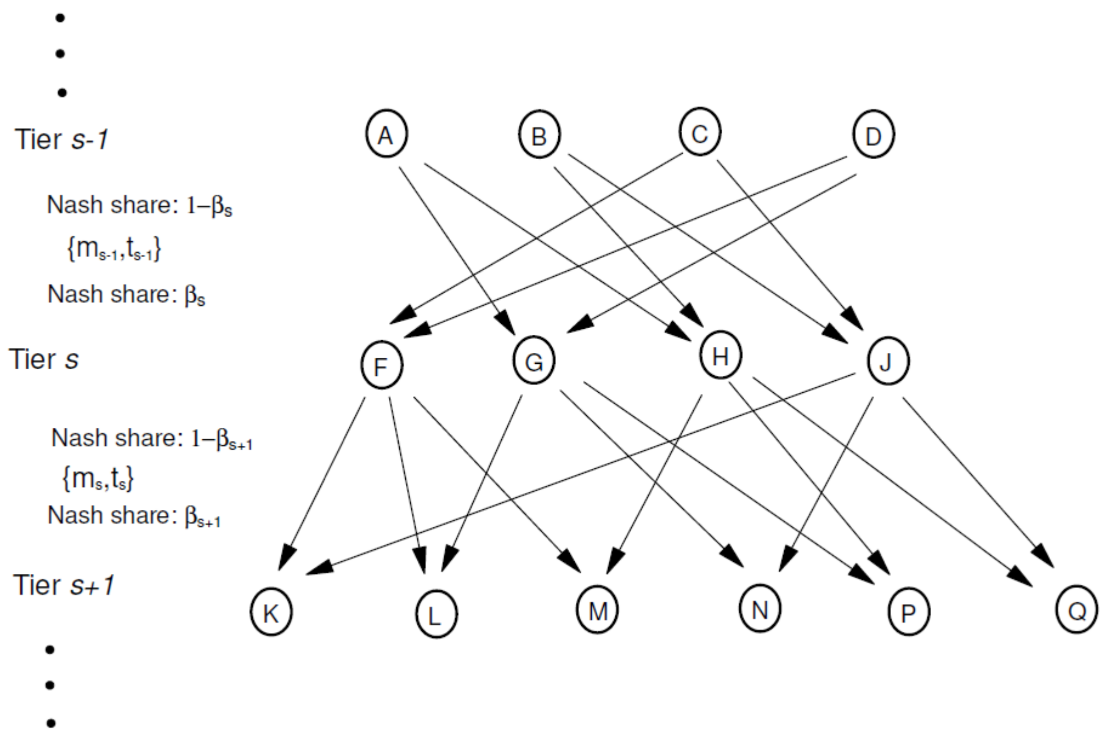

Figure 2: Supplier Contracts and Relationships

their differentiated products in a monopolistically-competitive market at price p; these variables also depend on firm productivity in the extended model.

We proceed in the following sections to analyze the stages of the model in reverse order. We specify the preferences and production technologies and describe the unique equilibrium, beginning with production of final goods, followed by production of inputs, sequential bargaining between suppliers and buyers, and finally investments in protective capabilities and relationship links. In Section 2.10 we spell out the remaining condition for a general equilibrium in an economy with an inelastic labor supply, L. Throughout, we take the wage rate as numeraire.[10]

2.2 Production and Sale of Consumer Goods

Consumers hold preferences defined over all differentiated final goods, with a constant elasticity of substitution $\varepsilon > 1$ between every pair of products. Each of the $\phi_S(r_S)N_S$ surviving lead producers faces a demand with constant elasticity $-\varepsilon$ and a "demand shifter" $AP^{-\varepsilon}$ that is determined in general equilibrium.[11] With a continuum of final producers, each firm takes the demand shifter as given.

[10] As previously stated, we shall assume in the main text that all firms in a given tier have the same productivity. The appendix develops the equilibrium conditions for the more general case in which the productivities of surviving firms are drawn from tier-specific distributions, as in Melitz (2003).

[11] The demand shifter $A = Y/P^{-\varepsilon}$, where Y is aggregate real income and P is the aggregate price index of all differentiated consumer goods.

The typical firm produces output according to a Cobb-Douglas production function that combines labor and a bundle of intermediate inputs, with cost shares γ_S and $1-\gamma_S$, respectively. The input bundles comprise CES aggregates of the various inputs that firms have contracted to purchase, with elasticity of substitution $\sigma_S > 1$ between every pair. We write

$$x_S = l_S^{\gamma_S}\left[\int_{i\in\Omega_{S-1}^u} m_{S-1}(i)^{\alpha_S}\,di\right]^{\frac{1-\gamma_S}{\alpha_S}} \tag{1}$$

where $m_{S-1}(i)$ is the agreed quantity that the firm buys from supplier i in tier $S-1$, Ω_{S-1}^u is the set of surviving suppliers in that tier, and $\alpha_S \equiv (\sigma_S - 1)/\sigma_S$.[12]

The market demand implies $p = (x_S/A)^{-1/\varepsilon}$. The typical firm has n_S^u surviving suppliers in tier $S-1$, where $n_S^u = \eta_S \phi_{S-1}(r_{S-1}) N_{S-1}$ is the product of the number of relationships it has formed and the survival rate. It has negotiated deals to purchase m_{S-1} units of a differentiated input from each of its suppliers and to pay t_{S-1} to each one. Therefore, the firm chooses l_S at the production stage to maximize

$$\pi_S = A^{\frac{1}{\varepsilon}} l_S^{\frac{\gamma_S(\varepsilon-1)}{\varepsilon}} (m_{S-1})^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}} (n_S^u)^{\left(\frac{1-\gamma_S}{\alpha_S}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)} - l_S - n_{S-1}^u t_{S-1}, \tag{2}$$

the difference between revenues from the sale of x_S units and total production costs.

2.3 Production of Inputs

A firm in a middle tier $s \in \{1,\ldots,S-1\}$ produces with a Cobb-Douglas technology that combines labor and input bundles, with shares γ_s and $1-\gamma_s$; i.e.,

$$x_s = l_s^{\gamma_s}\left[\int_{i\in\Omega_s^u} m_{s-1}(i)^{\alpha_s}\,di\right]^{\frac{1-\gamma_s}{\alpha_s}}, \tag{3}$$

where Ω_s^u is the set of its surviving suppliers and $m_{s-1}(i)$ is the quantity purchased from supplier i. The differentiated inputs in its bundle bear a constant elasticity of substitution $\sigma_s > 1$, where $\sigma_s = 1/(1-\alpha_s)$. In equilibrium, the firms in tier s have agreed to supply m_s units of their output to each of n_s^d customers. The Cobb-Douglas technology dictates how much labor they must hire to fulfill their various sales contracts in the light of their various purchase contracts. By inverting the production function with output $x_s = n_s^d m_s$, we find

$$l_s = \left[\frac{n_s^d m_s}{\left(\int_{i=0}^{n_s^u} m_{s-1}(i)^{\alpha_s}\,di\right)^{\frac{1-\gamma_s}{\alpha_s}}}\right]^{\frac{1}{\gamma_s}} \quad \text{for } s\in\{1,\ldots,S-1\}, \tag{4}$$

[12] In the extended model in the appendix that allows for firm heterogeneity, the right-hand side of (1) is preceeded by z, an index of the productivity of the particular lead producer. The same is true for the production functions for goods in middle tiers and in the initial tier, which appear in (3) and (5) below.

where $\int_{i=0}^{n_s^u} m_{s-1}(i)^{\alpha_s} di = n_s^u (m_{s-1})^{\alpha_s}$ in the symmetric equilibrium that arises when productivities are homogeneous.

The firms in tier 0 produce using labor alone, with constant returns to scale. Choosing units so that one unit of labor generates one unit of output, we have

$$x_0 = l_0. \tag{5}$$

These firms have agreed to provide m_0 units to each of their n_0^d clients. In order to fulfill its contracts, a typical tier-0 producer must employ a workforce of

$$l_0 = n_0^d m_0. \tag{6}$$

2.4 Bargaining between a Buyer in Tier 1 and a Supplier in Tier 0

Turning to the procurement stages, we begin with the last set of negotiations, those between buyers in tier 1 and their suppliers in tier 0. A typical firm in tier 1 has committed to supply m_1 units of its product to each of its measure n_1^d of downstream customers. It takes as given its agreement to purchase m_0 units of inputs from each of a measure n_1^u of suppliers other than the (infinitesimal) one with whom it now negotiates. The bargaining takes place over a quantity $\tilde{m}_0$ and a payment $\tilde{t}_0$. If the negotiation fails, the downstream firm must do without this marginal input. Instead, it would need to hire a small amount of additional labor to fulfill its own contracts. The firm's surplus from the relationship with the particular seller amounts to the savings in labor cost less the extra payment. We denote this surplus by $\psi_1^d(\tilde{m}_0, \tilde{t}_0)$.

In the appendix, we calculate the labor-cost savings by differentiating l_1 in (4) with respect to n_1^u (the measure of upstream suppliers) and evaluating the derivative at $\tilde{m}_0$, the quantity provided by the marginal supplier when all other suppliers provide m_0. Then we take $\psi_1^d(\tilde{m}_0, \tilde{t}_0) = -\partial l_1(\tilde{m}_0; m_0)/\partial n_1^u - \tilde{t}_0$.[13]

Meanwhile, the supplier in tier 0 stands to gain a payment of $\tilde{t}_0$ if it manages to strike a deal with the particular customer, but it would bear an extra labor cost of $\tilde{m}_0$ to produce the required output. The seller's surplus in a deal calling for $\tilde{m}_0$ and $\tilde{t}_0$ is simply $\psi_0^u(\tilde{m}_0, \tilde{t}_0) = \tilde{t}_0 - \tilde{m}_0$.

As usual, the Nash bargain solves

$$\{m_0, t_0\} = \arg \max_{\{\tilde{m}_0, \tilde{t}_0\}} \psi_1^d(\tilde{m}_0, \tilde{t}_0)^{\beta_1} \psi_0^u(\tilde{m}_0, \tilde{t}_0)^{1-\beta_1},$$

where β_1 is the bargaining weight of the buyer and $1-\beta_1$ is that of the seller. In the appendix, we

[13] Specifically, we find

$$\psi_1^d(\tilde{m}_0, \tilde{t}_0) = \frac{1-\gamma_1}{\alpha_1\gamma_1} [l_1(m_0)]^{\frac{1-\gamma_1(1-\alpha_1)}{1-\gamma_1}} \left(n_1^d m_1\right)^{\frac{-\alpha_1}{1-\gamma_1}} \tilde{m}_0^{\alpha_1} - \tilde{t}_0.$$

show that the first-order conditions for this maximization problem imply

$$m_0 = \left(\frac{1-\gamma_1}{\gamma_1}\right)^{\gamma_1} (n_1^u)^{\frac{\gamma_1-\sigma_1}{\sigma_1-1}} n_1^d m_1. \tag{7}$$

Intuitively, the negotiated quantity grows linearly with the volume of output, $n_1^d m_1$, that the tier-1 firm has promised to deliver to its downstream customers. The quantity m_0 falls with n_1^u, because a larger bundle of inputs into tier-1 production offers more substitutes for any particular one of them.

We also use the first-order conditions to calculate the negotiated payment, t_0, and find

$$t_0 = \mu_0 m_0$$

where

$$\mu_0 \equiv \beta_1 + (1-\beta_1)\frac{\sigma_1}{\sigma_1 - 1}.$$

Evidently, the total payment is proportional to the quantity, so μ_0 can be interpreted as a per-unit payment. If all of the bargaining power were to rest with the buyer ($\beta_1 = 1$), the per-unit payment would be $\mu_0 = 1$, which is the unit production cost. Alternatively, if all bargaining power were to rest with the seller ($\beta_1 = 0$), the per-unit payment would be $\mu_0 = \sigma_1/(\sigma_1 - 1)$, which is the monopoly price of a differentiated input when the elasticity of demand is σ_1. In general, the per-unit payment by a tier-1 producer is a weighted average of the competitive price of the input and the monopoly price, with the Nash-bargaining shares serving as weights.

We shall refer to μ_0 as a *markup factor,* by analogy to the pricing of differentiated inputs in an economy with monopolistic competition. Here, it measures the ratio of the negotiated payment to the supplier's production cost. The Nash bargaining protocol with a continuum of buyers and suppliers generates a constant "markup", which is greater when the seller has more bargaining power ($1-\beta_1$ is large) and when the seller's input substitutes poorly for other inputs used by the downstream customer (σ_1 is small).

2.5 Bargaining between a Buyer in Tier 2 and a Supplier in Tier 1

Next consider the negotiation between a typical buyer in tier 2 and a seller in tier 1. The downstream firm has committed to supply m_2 units to each of its n_2^d customers. It takes as given its agreement to purchase m_1 units of inputs from each of a measure n_2^u of other suppliers. Using (4) again, with $s = 2$, we can calculate the labor savings for the buyer from expanding its set of suppliers slightly and by purchasing $\tilde{m}_1$ units from the marginal seller. The surplus for the downstream firm, $\psi_2^d(\tilde{m}_1, \tilde{t}_1)$ is the difference between the marginal wage savings and the payment to the supplier, as before.

However, the calculation of the surplus for the seller is slightly different, because now the firms must anticipate subsequent negotiations, in keeping with the requirements for subgame perfection. The seller in tier 1 stands to gain the payment $\tilde{t}_1$ under the proposed contract. In order to fulfill such a contract, it will choose to hire marginally more labor. But it will also choose to purchase additional inputs from its other suppliers, which will necessitate a marginally larger bill for its input bundle. In the appendix, we calculate the marginal wage bill, $\partial l_1/\partial n_1^d$, and and the marginal input bill, $\partial\left(n_1^u t_1\right)/\partial n_1^d$, and evaluate both at $\tilde{m}_1$. We find that the extra cost of producing $\tilde{m}_1$ units for a marginal buyer amounts to $c_1\tilde{m}_1$, where c_1 is defined in (A.19) as

$$c_1 = \gamma_1^{-\gamma_1}\left(1-\gamma_1\right)^{-(1-\gamma_1)}\left(n_1^u\right)^{-\frac{1-\gamma_1}{\sigma_1-1}} B_1 \tag{8}$$

and

$$B_1 \equiv \gamma_1 + \left(1-\gamma_1\right)\mu_0. \tag{9}$$

We interpret c_1 as the marginal cost to a tier-1 producer of providing an additional unit of its input to one of its customers. The marginal cost decreases with n_1^u, because a more diverse set of tier-0 inputs makes its own input bundle more productive. The marginal cost increases with B_1, which is a cost-share weighted average of the wage and the anticipated, per-unit payment for inputs by the tier-1 supplier. Importantly, the marginal cost of producing tier-1 inputs grows with the markup μ_0 that the firm expects to emerge from its negotiations with its own suppliers.

Using the expressions for $\psi_2^d\left(\tilde{m}_1,\tilde{t}_1\right)$ and $\psi_1^u = \tilde{t}_1 - c_1\tilde{m}_1$, we can solve for the Nash bargain,

$$\{m_1,t_1\} = \arg\max_{\{\tilde{m}_1,\tilde{t}_1\}} \psi_2^d\left(\tilde{m}_1,\tilde{t}_1\right)^{\beta_2}\psi_1^u\left(\tilde{m}_1,\tilde{t}_1\right)^{1-\beta_2}.$$

In the appendix, we show that the first-order conditions imply

$$m_1 = c_1^{-\gamma_2}\left(\frac{1-\gamma_2}{\gamma_2}\right)^{\gamma_2}\left(n_2^u\right)^{\frac{\gamma_2-\sigma_2}{\sigma_2-1}} n_2^d m_2. \tag{10}$$

The solution implies that the typical seller in tier 1 delivers a smaller quantity of inputs to a typical customer when it perceives the marginal cost of producing those inputs to be higher. In other words, when a tier-1 seller and a tier-2 buyer choose the size of their transaction, they take account of the per-unit payment for tier-0 inputs that will result from the subsequent negotiations. Apart from this, (10) has the same form and interpretation as (7).[14]

We can also calculate the payment implied by Nash bargaining and find

$$t_1 = \mu_1 c_1 m_1, \tag{11}$$

where

$$\mu_1 \equiv \beta_2 + \left(1-\beta_2\right)\frac{\sigma_2}{\sigma_2-1}.$$

Here, $\mu_1 c_1$ is the per-unit payment that emerges from the negotiations between the tier 1 producer and the tier 2 producer. It is a (constant) markup μ_1 over the unit cost c_1, where the markup reflects

[14] The marginal cost of producing the tier-0 input is $c_0 = 1$.

the bargaining shares of the two sides and the substitutability of tier-1 inputs in the production function for x_2.

2.6 Bargaining between a Buyer in Tier *s (1<s<S)* and a Supplier in Tier *s-1*

We proceed in a similar fashion to solve for all of the remaining Nash bargains between non-extreme buyers and sellers. A typical supplier in tier $s-1$ sells a quantity

$$m_{s-1} = c_{s-1}^{-\gamma_s} \left(\frac{1-\gamma_s}{\gamma_s} \right)^{\gamma_s} \left(n_s^u\right)^{\frac{\gamma_s - \sigma_s}{\sigma_s - 1}} n_s^d m_s \tag{12}$$

to a typical buyer in tier s in exchange for a payment of

$$t_{s-1} = \mu_{s-1} c_{s-1} m_{s-1}, \tag{13}$$

where $\mu_{s-1} \equiv \beta_s + (1-\beta_s) \frac{\sigma_{s-1}}{\sigma_{s-1}-1}$ is the markup factor that results from negotiations between the firms in tier $s-1$ and tier s,

$$c_{s-1} = \prod_{j=1}^{s-1} \gamma_j^{-\gamma_j \Gamma_{j+1}^{s-1}} \left(1-\gamma_j\right)^{-\left(1-\gamma_j\right)\Gamma_{j+1}^{s-1}} \left(n_j^u\right)^{-\frac{\Gamma_j^{s-1}}{\sigma_j - 1}} \left(B_j\right)^{\Gamma_{j+1}^{s-1}} \tag{14}$$

is the unit cost of production for the firm in tier $s-1$, $\Gamma_j^s \equiv \Pi_{i=j}^s (1-\gamma_i)$ is the product of the input shares for all stages between j and s, and $B_j \equiv \gamma_j + \left(1-\gamma_j\right) \mu_{j-1}$ is defined analogously to B_1. We obtain equation (14) from (Axxx) in the appendix by utilizing the recursive structure of c_s.

The negotiated quantity m_{s-1} in (12) depends on the marginal production cost c_{s-1}, the measure of competing inputs n_s^u, and the total amount of downstream demand, $n_s^d m_s$, much as for m_1. But now the marginal cost reflects the diversity in the input bundles and the input-share weighted averages of the wage and the price of input bundles *in all stages further upstream*. The per-unit payment in (13) is the product of the marginal cost and a markup factor, μ_{s-1}, that emerges from the negotiation at hand.

Evidently, the per-unit payment by tier-s producers to their suppliers in tier $s-1$ reflects not only the division of surplus between the two negotiants, but also the markups they anticipate will emerge from bargaining further upstream. This outcome is the analog under sequential bargaining to the *double marginalization* that results from monopoly pricing of inputs in a market setting. With sequential bargaining, as with successive rounds of markup pricing, cost premia cumulate along the supply chain.

2.7 Bargaining between a Lead Firm and a Supplier in Tier *S-1*

Finally, we come to the negotiation between a typical final producer in tier S and a typical one of its suppliers in tier $S-1$. According to the sequencing outlined in Figure 1, these negotiations happen first, ahead of all the other bargaining. But they take place in anticipation of all that will follow.

The final producer expects to employ labor so as to maximize profits in (2). This gives the usual markup pricing over marginal cost, as in Dixit and Stiglitz (1977) and elsewhere. Substituting the resulting employment, l_S, into the expression for profits gives a relationship between profits net of labor costs, the size and productivity of the firm's input bundle, and the total payment to suppliers. Profits increase with the measure of input suppliers, all else the same, because the CES aggregator implies a love of input variety.

We can calculate the surplus of a lead producer in its relationship with one of its suppliers by taking the marginal gain in profits with respect to a marginal seller that provides input quantity $\tilde{m}_{S-1}$ and subtracting from this amount the payment $\tilde{t}_{S-1}$ to that marginal supplier. The marginal profit gain can be computed by differentiating π_S with respect to n_S^u and evaluating the quantity provided by the marginal firm at $\tilde{m}_{S-1}$. This gives $\psi_S^d(\tilde{m}_{S-1}, \tilde{t}_{S-1})$, as reported in (A.41).

As for the seller in this relationship, the calculus is the same as for any other supplier in a tier $s > 1$. The potential sale offers a gain of $\tilde{t}_{S-1}$ but at the expense of additional labor costs and additional input costs. The total additional costs are captured by $c_{S-1}\tilde{m}_{S-1}$.[15] The surplus is given by $\psi_{S-1}^u(\tilde{m}_{S-1}, \tilde{t}_{S-1}) = \tilde{t}_{S-1} - c_{S-1}\tilde{m}_{S-1}$. The Nash bargain, $\{m_{S-1}, t_{S-1}\}$ maximizes the geometric average of $\psi_S^d(\tilde{m}_{S-1}, \tilde{t}_{S-1})$ and $\psi_{S-1}^u(\tilde{m}_{S-1}, \tilde{t}_{S-1})$, with β_S and $1 - \beta_S$ as geometric weights.

The first-order conditions for the bargaining problem imply

$$m_{S-1} = A \left(c_{S-1}\right)^{\gamma_S(\varepsilon-1)-\varepsilon} \left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S(\varepsilon-1)} \left[\frac{\left(1-\gamma_S\right)\left(\varepsilon-1\right)}{\varepsilon}\right]^{\varepsilon} \left(n_S^u\right)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\sigma_{S-1}}} \tag{15}$$

and

$$t_{S-1} = \mu_{S-1} c_{S-1} m_{S-1}. \tag{16}$$

The lead producer buys more inputs from a typical supplier when aggregate demand for inputs (as captured by A) is great, when the perceived marginal cost of producing those inputs, c_{S-1}, is small and when inputs are productive thanks to their diversity. It negotiates a payment for its inputs that is a multiple $\mu_{S-1} = \beta_S + (1-\beta_S)\frac{\sigma_{S-1}}{\sigma_{S-1}-1}$ of the production costs.

2.8 Recursive Solution for Quantities, Payments, and Employment Levels

We can now use the various bargaining solutions to express the input quantities $\{m_{s-1}\}$, the payments $\{t_{s-1}\}$, and the employment levels $\{l_s\}$ as functions of the aggregate demand shifter A and the numbers of active input suppliers per firm $\{n_s^u\}$ in every tier. First, we eliminate from the equations the number of *customers* for a typical firm in tier $s-1$ using the fact that every transaction involves one customer and one supplier. The $\phi_{s-1}(r_{s-1})N_{s-1}$ active firms in tier $s-1$ each have n_{s-1}^d customers, which gives a total of $\phi_{s-1}(r_{s-1})N_{s-1}n_{s-1}^d$ customer relationships.

[15] Here, c_{S-1} can be calculated using the formula for c_{s-1} in (14).

Meanwhile, the $\phi_s(r_s)N_s$ active firms in tier s each have n_s^u suppliers, for a total of $\phi_s(r_s)N_s n_s^u$ supply relationships. Since each customer relationship corresponds to one supply relationship, we have $\phi_{s-1}(r_{s-1})N_{s-1}n_{s-1}^d = \phi_s(r_s)N_s n_s^u$, or

$$n_{s-1}^d = \frac{\phi_s(r_s)N_s}{\phi_{s-1}(r_{s-1})N_{s-1}} n_s^u.$$

Now we solve the system of equations for $\{m_s\}$ recursively. We use (16) to solve for m_{S-1} as a function of A and the numbers of suppliers per firm in tiers S and above.[16] Then, given any m_s and the numbers of suppliers per firm in tier s and above, we use (12) to solve for m_{s-1}. Finally, given m_1 and the number of suppliers to firms in tier 1, we use (7) to solve for m_0.

Once we have all of the input quantities, we use (11), (13), and (16) to solve for the payments for each transaction and the (inverted) production functions (4) and (6) to solve for the employments levels.[17]

2.9 Protective Capabilities and Network Thickness

We turn finally to the initial stage of the game, when firms choose their protective capabilities and those in tier 1 and beyond form their supply networks.[18] We consider first the problem facing a firm in tier $s > 0$ that takes the investment decisions of all other firms as given. The firm in question chooses $\tilde{r}_s$ and $\tilde{\eta}_s$ to maximize its expected net profits[19],

$$v_s(\tilde{r}_s, \tilde{\eta}_s) = \phi(\tilde{r}_s)\, \pi_s(\tilde{\eta}_s; \boldsymbol{\eta}, \boldsymbol{r}) - \tilde{r}_s - k\tilde{\eta}_s N_{S-1}$$

where $\pi_s(\cdot)$ denotes the firm's operating profits *conditional on avoiding a supply disruption and* $\tilde{r}_s + k\tilde{\eta}_s N_{s-1}$ represents the total costs of its investments in resilience.

Notice that, conditional on survival, a firm's prior investment in protective capability has no influence on its operating profits. A firm in any tier s (including $s = 0$) chooses $\tilde{r}_s$ to maximize $v_s(\tilde{r}_s, \tilde{\eta}_s)$, which gives the first-order condition

$$\phi'(\tilde{r}_s)\, \pi_s(\tilde{\eta}_s) = 1. \tag{17}$$

Naturally, investments in protective capabilities are larger when the prospective profits for operating are greater.

The thickness of a firm's network does affect its subsequent operating profits, because it determines the variety of its inputs after supply shocks are realized. This, in turn, determines the firm's productivity and thus the outcomes in its negotiations with suppliers and customers. The

[16] The number of suppliers per firm, n_s^u, for all $s \leq S-1$ figure in the expression for c_{S-1}.

[17] We also need the first-order condition for profit maximization by final producers to solve for l_S.

[18] A firm in tier 0 faces a similar problem when choosing its protective capabilities, r_0, but it has no relationships with input suppliers.

[19] In the appendix, where we admit heterogeneity in ex-post productivity, v_s is the expected value of net profits over possible realizations of productivity z.

first-order condition for the choice of $\tilde{\eta}_s$ can be written as

$$\phi\left(\tilde{r}_s\right)\pi_s'\left(\tilde{\eta}_s\right) = kN_{S-1}. \tag{18}$$

Clearly, we need to derive $\pi_s'\left(\tilde{\eta}_s\right)$, the marginal effect of a thicker network on a firm's operating profits.

Consider a firm in a middle tier, i.e., $s \in \{1, 2, \ldots, S-1\}$. The firm's operating profits are the difference between its receipts from all downstream customers and its total production costs. Production costs comprise the sum of payments to all suppliers and the firm's wage bill. We write

$$\pi_s\left(\tilde{\eta}_s\right) = n_s^d t_s(\tilde{\eta}_s) - n_s^u\left(\tilde{\eta}_s\right) t_{s-1}\left(\tilde{\eta}_s\right) - l_s\left(\tilde{\eta}_s\right).$$

The number of a firm's supplier links has no bearing on the size of its customer base, n_s^d, which is determined by decisions of downstream firms. But more links means more surviving suppliers and having more suppliers spells higher productivity. With higher productivity, the firm achieves a lower unit cost and sells more to each of its customers. It receives a payment per customer of $t_s\left(\tilde{\eta}_s\right) = \mu_s \tilde{c}_s\left(\tilde{\eta}_s\right)\tilde{m}_s\left(\tilde{\eta}_s\right)$. Notice that $\mu_s \equiv \beta_{s+1} + \left(1-\beta_{s+1}\right)\frac{\sigma_{s+1}}{\sigma_{s+1}-1}$ depends on the bargaining weight of the firm vis-à-vis its customers and the elasticity of substitution between the firm's output and that of other suppliers to the same buyer. Neither of these depends on the thickness of a firm's own supplier network. But $\tilde{c}_s\tilde{m}_s$ grows at a constant rate with $\tilde{\eta}_s$, because the firm negotiates larger sales to each of its customers, who substitute its product for other inputs to take advantage of their lower cost.

Meanwhile, the firm's total costs rise with $\tilde{\eta}_s$, because the firm makes larger commitments to its customers. We find that production costs also increase at a constant rate as the number of supplier links grows.

In the appendix, we show in (Axxx) that

$$\pi_s\left(\tilde{\eta}_s\right) = Q_{\pi_s}\tilde{\eta}_s^{\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s-1}}, \tag{19}$$

where Q_{π_s} is a constant from the firm's point of view. The elasticity of expected profits with respect to the firm's investment in relationship links is greater when having a more diverse set of inputs contributes more to productivity, i.e., when inputs are a larger share of production costs for firms in tier s (higher $1-\gamma_s$) and when the inputs used by these firms are more differentiated (smaller σ_s). A given productivity gain is more beneficial to a firm in tier s when its competitors produce inputs that are closer substitutes for its own in the eyes of its downstream customers (higher σ_{s+1}).

The power function on the right-hand side of (19) reflects the CES technology for the input bundle and the Cobb-Douglas combination of inputs and labor. Indeed, the profit elasticity here is reminiscent of that in settings with monopolistically competitive input markets. Although our prices and quantities result from sequential bargaining in a complex supply chain, the mechanism by which input variety raises profits is similar to what happens in a setting with unilateral price setting. In a model with monopolistic competition and CES technology, an increase in the number of inputs makes the inputs more productive, while leaving markups unchanged. With greater productivity and unchanged prices, a firm sells more inputs and earns greater profits. Here, firms negotiate with each of their customers and then with their suppliers. An increase in productivity has no effect on the negotiated "markups," but it does increase the profits that can be shared in each pairwise negotiation. A more productive firm negotiates a larger volume of sales with each of its customers and larger purchases from each of its suppliers, which generates increased profits all along its supply chain.

We can use a similar procedure to find how π_S, the operating profits of a final producer in (2), vary with the firm's investment in supply links. We need to calculate how revenues and costs vary with $\tilde{\eta}_S$, which is tedious but straightforward. The calculations yield (see (Axxx))

$$\pi_S\left(\tilde{\eta}_S\right) = Q_{\pi S}\tilde{\eta}_S^{\frac{(1-\gamma_S)(\varepsilon-1)}{\sigma_S-1}}. \tag{20}$$

For interior solutions to the optimization problem in (18), we need that $\pi_s\left(\tilde{\eta}_s\right)$ and $\pi_S\left(\tilde{\eta}_S\right)$ are concave functions. Concavity of these functions is ensured by the following assumption.

Assumption 1 $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_S \geq \varepsilon.$

Assumption 1 says that a good becomes more and more differentiated as it proceeds down the supply chain. This seems a reasonable assumption about the multi-stage transformation of raw materials into ever-more-customized inputs and finally into consumer products.

2.10 General Equilibrium

A labor-market clearing condition closes the model. Labor is used to produce intermediate inputs, to produce final goods, to form supply networks, and to acquire protective capabilities at every level in the supply chain. Production labor in a typical firm in tier s must satisfy (4) for $s \in \{1, \ldots, S-1\}$ and (6) for $s = 0$. Final producers hire labor l_S to maximize operating profits in (2). In addition, each firm in tier s employs r_s workers to protect against its own supply disruption and each firm in tier $s \neq 0$ employs $k\eta_s N_{s-1}$ workers to form supply relationships with firms upstream. There are $\phi_s\left(r_s\right) N_s$ active firms in tier s after the resolution of the supply shocks. Therefore, the general equilibrium requires

$$\sum_{s=0}^{S} N_s r_s + \sum_{s=1}^{S} N_s k\eta_s N_{s-1} + \sum_{s=0}^{S} \phi_s\left(r_s\right) N_s l_s = L.$$

This condition determines the demand shifter A that appears in (2) and (15); see (A.65) in the appendix and the discussion there.

3 First-Best Allocation and Optimal Policy

In this section, we characterize the optimal resource allocation in an economy with ongoing risks of supply disturbances. Then we identify the fiscal policies that would decentralize the first best as an equilibrium outcome. Although the informational requirements for implementing such policies would be severe, finding the optimal taxes and subsidies helps us to understand where inefficiencies can arise in a multi-tier supply chain.

The planner allocates resources to maximize welfare of the representative household. The constant-elasticity demand function facing each final producer derives, as usual, from a CES utility function,

$$W=\left[\int_{j\in\Omega_S} x_S\left(j\right)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where Ω_S is the set of differentiated products available to consumers. By symmetry, and with homogeneous production functions, the planner provides households with equal quantities x_S of all available consumer goods, so we can rewrite the planner's objective function as

$$W=\left(n_S\right)^{\frac{\varepsilon}{\varepsilon-1}}x_S, \tag{21}$$

where $n_S=\phi_S\left(r_S\right)N_S$ is the measure of final producers that avoid supply disturbances.[20]

With homogeneous production technologies, the symmetry of (3) also dictates that equal quantities m_s be provided to a typical producer in tier $s+1$ by every one of its input suppliers, considering the relationships it has formed and the suppliers that survive. A typical final producer has $n_S^u=\eta_S\phi_{S-1}\left(r_{S-1}\right)N_{S-1}$ suppliers. So, (1) implies $x_S=l_S^{\gamma_S}\left(m_{S-1}\right)^{1-\gamma_S}\left[\eta_S\phi_{S-1}\left(r_{S-1}\right)N_{S-1}\right]^{\frac{1-\gamma_S}{\alpha_S}}$. Then, substituting for x_S in (21), we can write the planner's problem as choosing all investments in protective capabilities, $\{r_s\}$, the thickness of all supply networks, $\{\eta_s\}$,the input quantities, $\{m_s\}$, and the manufacturing employment levels $\{l_s\}$ to maximize

$$W=\left[\phi_S\left(r_S\right)N_S\right]^{\frac{\varepsilon}{\varepsilon-1}}l_S^{\gamma_S}\left(m_{S-1}\right)^{1-\gamma_S}\left[\eta_S\phi_{S-1}\left(r_{S-1}\right)N_{S-1}\right]^{\frac{1-\gamma_S}{\alpha_S}} \tag{22}$$

subject to the various resource constraints. First, labor employed in all uses should not exceed the inelastic supply, or

$$\sum_{s=0}^{S}N_sr_s+\sum_{s=1}^{S}N_sk\eta_sN_{s-1}+\sum_{s=0}^{S}\phi_s\left(r_s\right)N_sl_s\leq L. \tag{23}$$

Second, the m_s units of inputs provided to the $\phi_{s+1}\left(r_{s+1}\right)N_{s+1}$ downstream producers by each of their $\eta_{s+1}\phi_s\left(r_s\right)N_s$ suppliers in tier s should not exceed the aggregate amount of tier-s inputs

[20] As with the market equilibrium, we solve the planner's problem in the appendix allowing for Hicks-neutral productivity differences in all tiers of the supply chain.

produced, or

$$\left[\phi_{s+1}\left(r_{s+1}\right)N_{s+1}\right]\left[\eta_{s+1}\phi_{s}\left(r_{s}\right)N_{s}\right]m_{s}\leq\phi_{s}\left(r_{s}\right)N_{s}l_{s}^{\gamma_{s}}\left(m_{s-1}\right)^{1-\gamma_{s}}\left[\eta_{s}\phi_{s-1}\left(r_{s-1}\right)N_{s-1}\right]^{\frac{1-\gamma_{s}}{\alpha_{s}}}, \quad \text{for } s\in 1,\ldots,S-1, \tag{24}$$

where we have taken into account the Cobb-Douglas technology (3) available to each of the $\phi_s(r_s)N_s$ suppliers. Finally, the planner must not allocate more of the tier-0 input than is produced by the $\phi_0(r_0)N_0$ surviving firms, or

$$\left[\phi_{1}\left(r_{1}\right)N_{1}\right]\left[\eta_{1}\phi_{0}\left(r_{0}\right)N_{0}\right]m_{0}\leq\phi_{0}\left(r_{0}\right)N_{0}l_{0}, \tag{25}$$

in the light of the linear technology described by (6).

In the optimal allocation, the constraints are satisfied with equality. The first-order conditions with respect to labor l_s for all $s\in\{0,\ldots,S\}$ and input quantities m_s for all $s\in\{0,\ldots,S-1\}$ dictate that the ratio $l_s^*/n_s^u m_{s-1}^*$ of labor to aggregate inputs employed by a firm in tier s, $s\in\{1,\ldots S\}$, should equal $\frac{\gamma_s}{1-\gamma_s}\frac{\rho_{s-1}}{\omega}$, where ρ_s denotes the shadow value of a tier s input (the Lagrange multiplier on constraint (24) or (25), as the case may be), and ω denotes the shadow value of labor (the Lagrange multiplier on constraint (23)); this is the usual relationship between optimal cost shares that results from the Cobb-Douglas technology. Also, $\rho_0=\omega$, because the planner can readily convert one unit of labor into one input of a tier-0 input. Therefore,

$$\frac{l_1^*}{n_1^u m_0^*}=\frac{\gamma_1}{1-\gamma_1}. \tag{26}$$

where asterisks indicate first-best allocations.

Next, we can use the optimal input cost share in tier 1, $\rho_0 n_1^u m_0^*=(1-\gamma_1)\rho_1 n_1^d m_1^*$, and the fact that $\rho_0=\omega$, to derive

$$\frac{l_2^*}{n_2^u m_1^*}=\gamma_1^{-\gamma_1}\left(1-\gamma_1\right)^{-(1-\gamma_1)}\frac{\gamma_2}{1-\gamma_2}\left(n_1^u\right)^{-\frac{1-\gamma_1}{\sigma_1-1}}, \tag{27}$$

where we have used the ratio of the optimal cost shares in tier 2, the relationship between $n_1^d m_1^*$ and (m_0^*,l_1^*) implied by the production function (4), and the value of $l_1^*/n_1^u m_0^*$ that has been solved in (26). The right-hand side of (27) represents the ratio of the Cobb-Douglas exponents in the production of tier-2 goods, adjusted for the productivity of the tier-1 inputs that reflects their variety. Proceeding similarly and recursively, we can compute the optimal input ratios $l_s^*/n_s^u m_{s-1}^*$ for $s\in\{3,\ldots,S\}$ using $\rho_{s-1}n_s^u m_{s-1}^*=(1-\gamma_s)\rho_s n_s^d m_s^*$ and the relationship between output $n_s^d m_s^*$ and inputs $\left(l_s^*,m_{s-1}^*\right)$ that is implied by (4). This gives us the optimal allocations of labor, $\{l_s^*\}_{s=0}^{S}$ and the optimal input quantities, $\{m_s^*\}_{s=0}^{S-1}$, for any numbers of active upstream and downstream relationships, $\left\{n_s^d\right\}_{s=0}^{S-1}$ and $\{n_s^u\}_{s=1}^{S}$.[21]

[21] Using the solutions for l_S^* and m_{S-1}^*, we can then recover the optimal sales of a typical final good, x_S^*, from the production function.

The first-best numbers of supply relationships at every tier result from optimal investments in protective capabilities and optimal investments in supplier links. In the appendix, we show that the first-order conditions with respect to η_s, l_s and m_{s-1} together imply (see (Axxx) and (Axxx))

$$\frac{kN_sN_{s-1}\eta_s^*}{L-\sum_{j=0}^S N_j r_j^* - \sum_{j=1}^S kN_{j-1}N_j\eta_j^*} = \frac{\Gamma_s^S}{\sigma_s - 1} \qquad \text{for } s = \{1,2,\ldots,S-1\}\,, \tag{28}$$

and

$$\frac{kN_SN_{S-1}\eta_S^*}{L-\sum_{j=0}^S N_j r_j^* - \sum_{j=1}^S kN_{j-1}N_j\eta_j^*} = \frac{1-\gamma_S}{\sigma_S - 1},$$

where we recall that $\Gamma_j^s \equiv \Pi_{i=s}^S (1-\gamma_i)$ denotes the product of the input shares for stages s and beyond. The left-hand side of (28) is the ratio of the aggregate amount of labor optimally used for forming supplier links to the aggregate labor optimally used in manufacturing inputs and final goods. The right-hand side of (28) reflects the cumulation of cost shares beginning with tier s and the elasticity of substitution between inputs used in that tier. The greater are the input shares downstream and the less substitutable are the inputs used in tier s, the more socially valuable are links to suppliers in tier $s-1$.

As for the optimal investments in protective capabilities, we combine the first-order conditions with respect to r_s with the conditions for the optimal quantities, and find (see (Axxx) and (Axxx) in the appendix)

$$\frac{N_s r_s^*}{L-\sum_{j=0}^S N_j r_j^* - \sum_{j=1}^S kN_{j-1}N_j\eta_j^*} = \frac{\Gamma_{s+1}^S}{\sigma_{s+1} - 1}\frac{\phi_s'(r_s^*)\, r_s^*}{\phi_s(r_s^*)} \qquad \text{for } s = \{0,1,\ldots,S-1\} \tag{29}$$

and

$$\frac{N_S r_S^*}{L-\sum_{j=0}^S N_j r_j^* - \sum_{j=1}^S kN_{j-1}N_j\eta_j^*} = \frac{1}{\varepsilon - 1}\frac{\phi_S'(r_S^*)\, r_S^*}{\phi_S\left(r_S^*\right)}. \tag{30}$$

In both (29) and (30), the left-hand side is the ratio of the aggregate labor optimally used to promote firm survival in some tier to the aggregate labor optimally used for manufacturing, while the right-hand side reflects the social benefits of survival at that tier. In all tiers, the benefits increase with the elasticity of survival probability with respect to investment. For intermediate goods, they also increase with the cost shares of intermediates in all tiers downstream from s and decrease with the elasticity of substitution between tier-s inputs when used in tier $s+1$; firm survival is more valuable when inputs comprise a greater share of costs along the supply chain and when the inputs are imperfect substitutes. The survival of final-good producers is socially more valuable when their outputs are less substitutable in the eyes of consumers.

Finally, we are ready to compare the equilibrium allocation described in Section 2 with the first-best allocation described immediately above. To do so, we introduce three sets of policies that would allow the planner to implement the first-best allocation as a decentralized equilibrium.[22]

[22] The private and social incentives for resource allocation diverge on three margins, for m_s, r_s, and η_s. Therefore,

These policies represent "wedges" between private and social incentives for each use of resources. We let $\{\tau_s\}_{s=0}^{S-1}$ be the sequence of sales policies along the supply chain, where τ_s denotes the fraction of the cost of a tier-s input optimally paid by the downstream firm in tier $s+1$. Clearly, $\tau_s < 1$ represents a *subsidy* to promote sales from tier s to tier $s+1$, whereas $\tau_s > 1$ represents a *tax* to discourage such sales. Similarly, we let $\{\theta_s\}_{s=0}^{S}$ be the sequence of investment policies, where θ_s is the fraction (or multiple) of any investment aimed at avoiding supply disruptions that is paid by the firms in tier s. Finally, we let $\{\vartheta_s\}_{s=1}^{S}$ denote the sequence of policies directed at network formation, where ϑ_s denotes the fraction (or multiple) of the cost paid by a typical tier-s producer when forming links to potential suppliers in tier $s-1$. We assume that all subsidies are financed by lump-sum taxation, while tax revenues are rebated similarly. We discuss each of the wedges in turn.

3.1 Optimal Policies to Promote First-Best Input Transactions

Consider first the size of transactions between firms in tier 0 and tier 1. In the Nash-in-Nash bargaining solution, a pair of negotiants choose m_0 to maximize their joint surplus, taking as given the quantities in other relationships. When the downstream firm pays only the fraction τ_0 of what the upstream firm receives, the Nash bargain in (7) must be amended to read

$$m_0 = \left(\frac{1-\gamma_1}{\gamma_1\tau_0}\right)^{\gamma_1} [n_1^u]^{\frac{\gamma_1-\sigma_1}{\sigma_1-1}} n_1^d m_1.$$

Then, using the technological constraints in (4) and (6), this implies

$$\frac{l_1}{n_1^u m_0} = \frac{\gamma_1}{1-\gamma_1}\tau_0. \tag{31}$$

Now compare the left-hand side of (31), which is the equilibrium ratio of labor to intermediate inputs in a tier-1 firm, to the optimal ratio expressed in (26). We see that the social planner can implement the first-best transactions between these firms with $\tau_0^* = 1$, i.e., by keeping hands off.

Why are private and social incentives aligned for these transactions between the farthest-upstream firms? With sequential bargaining, the negotiations between tier-0 firms and tier-1 firms are the last to occur. A deal that emerges at this stage does not affect any other transactions. Since the outcome of this bargaining generates no externalities, what remains is a desire for joint efficiency in production, which the firms share with the social planner. Put differently, when the most upstream firms bargain, the potential surplus for the pair reflects the private marginal cost of producing the tier-0 input. But the private marginal cost mirrors the social marginal cost, because only labor is used in its production. It follows that the planner need not intervene in these upstream transactions.

Next, consider the private incentives in a transaction between a tier-1 firm and a tier-2 firm.

three policy instruments are necessary and sufficient to implement the first-best allocation.

The joint-surplus maximization in the Nash bargaining implies

$$\frac{l_2}{n_2^u m_1} = \frac{\gamma_2}{1-\gamma_2} c_1 \tau_1, \tag{32}$$

where we recall that c_1 is the marginal cost of a unit of the tier-1 input, including both the labor cost and the cost of acquiring the tier-0 input bundle. The left-hand side of (32) represents the ratio of physical quantities of labor to produced inputs in tier-2 production, while the right-hand side is the product of the ratio of the per-unit (shadow) factor costs and the optimal factor shares implied by the Cobb-Douglas technology. Using the expression for c_1 in (8), we see that the planner must intervene in these transactions to induce the efficient techniques. The efficient factor ratio requires $B_1 \tau_1^* = 1$, or

$$\tau_1^* = \frac{1}{\gamma_1 + (1-\gamma_1)\,\mu_0} < 1.$$

The optimal subsidy on sales of tier-1 inputs to tier-2 producers reveals a divergence between private and social incentives. In the absence of any policy, the pair will negotiate based on an anticipated *private* marginal cost of producing the tier-1 input that reflects the markup that will result subsequently when the tier-1 firm purchases inputs from its tier-0 suppliers. As we have noted, $\gamma_1 + (1-\gamma_1)\,\mu_0$ measures how much this anticipated markup distorts the cost of producing tier-1 inputs. The inflated private cost would lead the two firms to transact too little. The optimal subsidy counteracts this distortion, ensuring that the parties consider the *social* cost of producing tier-1 inputs when they set their procurement contract.

The qualitative properties of the optimal subsidy are readily understood. First, the markup on the tier-0 input depends on the bargaining weights in the negotiations between these suppliers and their tier-1 buyers. The optimal subsidy to sales by a tier-1 firm decreases monotonically with its bargaining weight in its subsequent negotiations with its own suppliers. If $\beta_1 = 1$, for example, all of the bargaining power in the negotiation between firms in tier 0 and tier 1 resides with the downstream firm, and then $\mu_0 = 1$. In this case, $\tau_1^* = 1$, i.e., there is no subsidy to transactions between firms in tier 1 and tier 2. The optimal subsidy $1-\tau_1^*$ declines with the elasticity of substitution between tier-0 inputs in producing tier-1 goods, because greater substitutability between these inputs weakens the bargaining position of the suppliers and so reduces the markup. The optimal subsidy falls with the labor share of cost in producing the tier-1 inputs, because a higher γ_1 implies that a given markup of input prices has a smaller impact on the marginal cost of m_1.

In Section A.4 of the appendix, we show that[23]

$$\tau_s^* = \frac{1}{\gamma_s + (1-\gamma_s)\,\mu_{s-1}} < 1, \text{ for all } s \in \{1, \ldots, S-1\}. \tag{33}$$

The logic for all of the subsidies is similar; in each negotiation, the private parties in tiers s and

[23] In fact, we show that (33) gives the optimal subsidy even when firms in a tier are heterogeneous in their productivities.

$s+1$ face a distorted marginal cost of the good they are transacting, because the producer of this good anticipates paying an elevated price for its own inputs in its subsequent negotiations. At each stage, the planner offsets the anticipated markup, thereby ensuring that the firms in s and $s+1$ choose the efficient quantities.

If all negotiations give similar weight to the relatively upstream firm and all inputs have similar production technologies, then all subsidies for tiers $s \geq 1$ will be the same. Alternatively, if inputs become more specialized (and thus strictly less substitutable) as a good proceeds down the supply chain (so that μ_{s-1} rises with s), and if bargaining weights and labor shares are equal all along the chain, then the optimal transaction subsidies rise monotonically as we move downstream.

Finally, the planner eschews any subsidy or tax on sales of the final good; $\tau_S^* = 1$. Although the lead producers charge prices in excess of their marginal costs, the markups are common to all final goods and so do not distort any consumption decisions. We summarize in

Proposition 1 *To achieve the first best, the planner subsidizes sales by all firms in intermediate tiers $s = \{1, 2, \ldots, S-1\}$. The optimal subsidy for any good depends only on parameters describing the technology for producing that good and on the bargaining weight of the producer when it negotiates with its suppliers. The planner neither subsidizes nor taxes sales by firms in the extreme ends of the supply chains.*

3.2 Optimal Policies to Promote First-Best Protective Capabilities

Next we compare the private and social incentives for investments in protective capabilities. We identify two conflicting forces that drive a wedge between the two. On the one hand, a firm in tier s garners only the fraction $1 - \beta_{s+1}$ of the joint surplus in its relationship with customers in tier $s+1$. The smaller is this share, the smaller is the firm's incentive to invest in protective capabilities. The planner, in contrast, is concerned with the total surplus, not the division between the parties. Thus, the surplus sharing tends to generate underinvestment in protective capabilities by firms all along the supply chain. On the other hand, the planner applies optimal subsidies to sales for all $s \in \{1, 2, \ldots, S-1\}$. These subsidies artificially raise profitability for the input buyers, which tends to incentivize investments in protective capabilities beyond their social value.

In keeping with this intuition, we derive a simple expression in (A.123) for the optimal policy toward investments in protective capabilities by firms producing inputs, namely

$$\theta_s^* = \frac{1 - \beta_{s+1}}{\tau_s^*} \text{ for all } s \in \{0, 1, \ldots, S-1\}. \tag{34}$$

First, notice that the optimal policy does not depend on properties of the function $\phi_s(r)$ that relates the probability of a disruption to the size of the investment. Although the elasticity of $\phi_s(r)$ affects the planner's preferred resilience (see (29)), that same elasticity also affects the firms' private incentives to avoid disturbances, and in much the same way. Second, the optimal policy depends only on the bargaining weight for the firm in its negotiations with its downstream customers and on the optimal subsidy on its purchases from its upstream suppliers. Since there is no subsidy for purchases of tier-0 inputs ($\tau_0^* = 1$), the planner always wishes to promote investment in protective capabilities in the most upstream tier of the supply chain ($\theta_0^* = 1 - \beta_1 < 1$). It might be that other far-upstream inputs are highly substitutable, in which case the transaction subsidies for these tiers will be small. Then, with τ_s^* close to one, the optimal policy promotes investment in protective capabilities in other upstream tiers as well. Further downstream, inputs may become more specialized and less substitutable. If the elasticity of substitution between inputs falls monotonically (and strictly) as the good moves downstream, and if bargaining weights and labor shares are similar along the chain, then the optimal subsidies for investment in protective capabilities will decline monotonically and may eventually turn from subsidy to tax. A tax on investments in protective capabilities will be indicated when a large markup of input costs must be offset by a large transaction subsidy, which inflates the incentives for survival greatly.

Turning to the protective capabilities of final producers, we find in (Axxx) that

$$\theta_S^* = 1 - \frac{(1 - \beta_S)(1 - \gamma_S)(\varepsilon - 1)}{\sigma_S - 1} < 1. \tag{35}$$

Since the sales by final producers are not subsidized (or taxed) in the first best, all that remains for the planner is to induce the producers to internalize the positive externalities for consumers generated by their presence in the marketplace. We have thus established

Proposition 2 *To achieve the first best, the planner subsidizes investments in protective capabilities at both extreme ends of the supply chain. For intermediate stages, the optimal policy depends only on parameters describing the technology for producing that good and on the bargaining weight of the producer when it negotiates with its customers. Under Assumption 1, if bargaining weights and labor shares are similar along the chain, then the optimal subsidies for investment in resilience decline monotonically. For some parameter values, it may be optimal to tax investments in protective capabilities in some tiers to offset the excessive private incentive induced by a large transaction subsidy.*

3.3 Optimal Policies to Promote First-Best Linkages

Similar considerations come into play when we consider the optimal policy toward network formation. On the one hand, firms in intermediate tier s tend to have insufficient incentive to form links with upstream suppliers, because they capture only a fraction of the surplus created by such investments. On the other hand, the sales by firms in tier s are subsidized, generating private profits that are not part of social surplus. These extra profits tend to incentivize excess investments in network formation.

To get a handle on whether subsidies to network formation ought to be bigger or smaller than those for investments in protective capabilities, let us compare the equilibrium ratio of investments in the two forms of resilience in the absence of policy with the ratio that maximizes social welfare. Concerning the private incentives, firms in tier s will invest more in relationships when the cost share of inputs is large (γ_s small), when diversity adds more to productivity (σ_s small), and when their own output substitutes more closely for that of their competitors (σ_{s+1} large), which allows them to steal more sales and profits from rivals following a reduction in cost. None of these parameters directly affects a firm's incentives to invest in protective capabilities, except inasmuch as they affect the level of operating profits. Using (17), (18) and the relationship between operating profits and network thickness in (19), we show in (Axxx) that

$$\frac{r_s}{\eta_s} = \frac{\sigma_s - 1}{(1-\gamma_s)(\sigma_{s+1}-1)} k N_{s-1} \varepsilon_\phi (r_s) . \tag{36}$$

where $\varepsilon_\phi (r_s) \equiv r_s \phi_s' (r_s) / \phi_s (r_s)$ is the elasticity of the survival probability with respect to the investment in protective capabilities.

The calculus for the social planner is seemingly different. The social benefits from relationship links for firms in tier s increase with the input share in tier s, but also with the input shares in all tiers downstream from s. And whereas imperfect substitutability of inputs used in tier s (σ_s small) raises the marginal social benefit from having additional suppliers, the substitutability between the inputs used in tier $s+1$ has no bearing on the marginal benefit, because the planner does not care about the distribution of profits among firms in tier s.[24] Meanwhile, the social benefit from investments in protective capabilities in tier s reflects the input share in tiers $s+1$ and beyond and they are larger when the tier-s inputs are less close substitutes for their customers. Dividing the first-order condition for r_s^* (29) by that for η_s^* (28), we find

$$\begin{aligned} \frac{r_s^*}{\eta_s^*} &= \frac{\Gamma_{s+1}^S}{\Gamma_s^S} \frac{\sigma_s - 1}{\sigma_{s+1}-1} \frac{k N_s N_{s-1}}{N_s} \varepsilon_\phi (r_s^*) \\ &= \frac{\sigma_s - 1}{(1-\gamma_s)(\sigma_{s+1}-1)} k N_{s-1} \varepsilon_\phi (r_s^*) . \end{aligned} \tag{37}$$

Notice that the expression on the second row of (37) is identical to that on the right-hand side of (36). Evidently, in the absence of any policy, the relative private incentives to invest in the alternative forms of resilience coincide with the social imperative. To preserve this equality in the presence of fiscal policies, the planner must subsidize (or tax) investments in protective capabilities and investments in network thickness *at the same rates*. It follows that the first-best policies for investments in relationship links satisfy

$$\vartheta_s^* = \frac{1-\beta_{s+1}}{\tau_s^*} \text{ for all } s \in \{0, 1, \dots, S-1\} . \tag{38}$$

The planner must also subsidize link formation by lead producers, with $\vartheta_S^* = \theta_S^*$, or

$$\vartheta_S^* = 1 - \frac{(1-\beta_S)(1-\gamma_S)(\varepsilon - 1)}{\sigma_S - 1}.$$

[24] In other contexts, the private incentive to enter or invest in order to capture profits at the expense of rivals has been termed the "business-stealing effect," and it generally tends to cause overinvestment relative to the social optimum.

We record

Proposition 3 *To achieve the first best, the planner levies a subsidy (or tax) on network formation at intermediate tier $s \in \{1, \ldots, S-1\}$ at the same rate as the optimal subsidy (or tax) on investments in protective capabilities. The planner subsidizes investments in network formation by final producers at the same rate as investments in protective capabilities.*

Admittedly, Proposition 3 relies on special features of our model. First, the Nash-in-Nash bargaining protocol generates constant "markups" that firms cannot manipulate by their choice of network thickness. Second, all firms in our model are small, so they cannot manipulate the general equilibrium in a way that improves their bargaining position vis-à-vis their suppliers or customers. Finally, the CES production technology creates a tight relationship between the positive externalities from investments in resilience that accrue to downstream customers and the negative externalities suffered by competing firms due to the loss of sales and profits. The offsetting "consumer-surplus" externality and "business-stealing externality" are familiar from other contexts with CES technologies (or preferences) and *ex ante* investments (in market entry or cost reduction).[25]

4 Second-Best Policies for Resilience

The salience of recent supply-chain disruptions has directed attention to what the government might do to promote greater chain resilience. In the current environment, policies that encourage firms to invest in reducing the likelihood of disruptions or in diversifying their input sources might be politically palatable even when direct subsidies to their sales are not. To address this apparent political reality, we consider in this section a second-best setting in which the government can subsidize investments in protective capabilities and network formation, but cannot bankroll firm-to-firm transactions along the supply chain.

The government's problem is the same as before, except that we impose $\tau_s = 1$ for all s. We denote by θ_s° the fraction of the cost of investing in protective capabilities paid by a firm in tier s, $s \in \{0, 1, \ldots, S\}$, in the second-best regime. Similarly, ϑ_s° is the share of the cost of network formation borne by a firm in tier s, $s \in \{1, 2, \ldots, S\}$.

In the appendix, we show in (Axxx)-(Axxx) that

$$\theta_s^\circ = \frac{1}{J} \left\{ \frac{1 - \beta_{s+1}}{\prod_{j=s+1}^{S-1} \left[\gamma_j + \left(1 - \gamma_j\right) \mu_{j-1}\right]} \right\} \qquad \text{for } s \in \{0, 1, \ldots, S-1\} \tag{39}$$

[25] See, for example, Tirole (1988, ch.7), Matsuyama (1995), Dhingra and Morrow (2019), and Matsuyama and Uschev (2021).

and

$$\theta_S^{\circ} = \frac{1}{J}\left[1 - \frac{(1-\beta_S)(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1}\right], \tag{40}$$

where $J \leq 1$ is a term that captures the aggregate labor-market effects of all the second-best policies.[26]

How do we understand the expressions for the second-best subsidies (or taxes) on investments in protective capabilities? The first thing to note is that, in the sequential bargaining equilibrium, the planner's objective, W, is multiplicatively separable in a term that depends on $\{r_s\}$ and $\{\eta_s\}$ and a term that reflects the sizes of the transaction subsidies, $\{\tau_s\}$ (see Axxx). This separability follows from the assumption of CES technologies and preferences, with their multiplicative aggregation properties. It implies that the planner targets the same investment levels no matter what are the transaction subsidies.[27] The second-best investment levels, $\{r_s^{\circ}\}$ and $\{\eta_s^{\circ}\}$, are the same as the first-best levels, $\{r_s^*\}$ and $\{\eta_s^*\}$ that are reported in (28), (29) and (30).

However, the private incentives for *ex ante* investments vary with the transaction policies, because these policies affect operating profits and thus the firms' incentives to invest in protective capabilities and network thickness. To achieve the *same investment levels*, $\{r_s^*\}$ and $\{\eta_s^*\}$, in a second-best equilibrium, the planner must impose *different policies* than those in (34) and (38).

As in the first-best setting, the planner must account for the positive externality associated with a firm's survival as a supplier. The upstream firm in every relationship captures only the fraction $1-\beta_{s+1}$ of the surplus from any investment in protective capabilities, while the remaining fraction β_{s+1} accrues to its customers. The second-best subsidies induce firms to invest based on the full surplus, rather than their negotiated shares. This externality accounts for the term $1-\beta_{s+1}$ in the numerator of (39), just as it figures in the first-best subsidy rate in (34).

However, the lack of transaction subsidies leaves in place the markups that distort tier-to-tier transactions. These distortions figure in the denominator of the term in the curly brackets in (39). The negotiated payments that exceed production costs reduce profitability at every stage; see (Axxx)-(Axxx). Consequently, they dim the incentives for investments in protective capabilities. In particular, since $B_j = \gamma_j + \left(1-\gamma_j\right)\mu_{j-1} > 1$ for all j, the denominators in the curly brackets all exceed one and thus contribute to even larger investment subsidies for every tier than are implied by the surplus sharing. But note that the uncorrected distortions do not affect profitability equally across tiers. Since the negotiated markups cumulate as we move downstream, the upstream firms lose more in sales and profits than do their counterparts downstream. This double marginalization points to the need for larger investment subsidies upstream than downstream.[28]

[26] Specifically,

$$J := \frac{\Gamma_1^S}{\prod_{j=1}^{S-1}\left[\gamma_j + (1-\gamma_j)\,\mu_{j-1}\right]} + \sum_{j=1}^{S-1}\frac{\gamma_j}{1-\gamma_j}\Gamma_j^S\prod_{z=j}^{S-1}\frac{1}{\left[\gamma_z + (1-\gamma_z)\,\mu_{z-1}\right]} + \gamma_S\ .$$

[27] If we can write $W(\mathbf{r},\boldsymbol{\eta},\boldsymbol{\tau}) = \tilde{W}(\mathbf{r},\boldsymbol{\eta})\,C_w(\boldsymbol{\tau})$, then $argmax_{\mathbf{r},\boldsymbol{\eta}}W(\mathbf{r},\boldsymbol{\eta},\boldsymbol{\tau})$ is independent of $\boldsymbol{\tau}$.

[28] The fact that $B_j > 1$ for all j implies that the denominator grows monotonically as we add more terms to the product.

Overall, the term in curly brackets suggests the desirability of second-best subsidies for investments in protective capabilities all along the supply chain. However, this conclusion may not be warranted when we consider the role of J. The term J captures the fact that the cost distortions collectively depress the demand for manufacturing labor. The resulting fall in the real wage raises profitability and incentives for *ex ante* investment. The smaller is J, the smaller are the second-best subsidies, and taxes may be needed in some downstream tiers to induce the socially-efficient investment levels.

We can readily compare the second-best subsidies at different points in the supply chain. Let us begin with second-best policy for investments in tier 0. We see that $J\Pi_{j=1}^{S-1}\left[\gamma_j + \left(1-\gamma_j\right)\mu_{j-1}\right] > 1$; i.e., the general-equilibrium effect of the subsidies cannot outweigh the strongest of the direct effects.[29] Since, with $s = 0$, the numerator in (39) is less than one and the denominator exceeds one, it follows that

$$\theta_0^{\circ} < 1;$$

i.e., in the second-best regime, it is always optimal for the government to subsidize investments in protective capabilities in the most upstream tier.

Turning to the relationship between the second-best subsidies in successive tiers, we have from (39) that

$$\frac{\theta_{s-1}^{\circ}}{\theta_s^{\circ}} = \frac{1-\beta_s}{1-\beta_{s+1}}\left[\frac{1}{\gamma_s + (1-\gamma_s)\,\mu_{s-1}}\right].$$

Thus, if $\beta_{s+1} \leq \beta_s$, then $\theta_{s-1}^{\circ} < \theta_s^{\circ}$; i.e. if bargaining weights are constant or decreasing along the supply chain, the second-best subsidies to investments in protective capabilities shrink as we proceed downstream. In the absence of transaction subsidies, and with $\beta_{s+1} \leq \beta_s$, the social imperative for resilience is greater for the upstream firm in any supplier-buyer relationship, due to the cumulation of cost distortions.

How do the second-best policies toward investments in protective capabilities compare with the first best? Both policies address the externality that results from rent sharing, as reflected in the bargaining weight, $1-\beta_{s+1}$. Beyond that, they address different distortions: excess private profitability created by transaction subsidies on the one hand, and contraction of downstream input demand caused by uncorrected markups on the other. As a result, these subsidies are not directly comparable. If the denominator of (39) exceeds one, as is mostly likely for firms that are far upstream, then $\theta_s^{\circ} < \theta_s^{*}$; i.e., the optimal second-best subsidy to resilience must exceed the first-best subsidy at tier s. This is a situation in which the downstream contraction of input demand caused by the successive markups leads to a substantial underinvestment in resilience in the absence

[29] Note that

$$\begin{aligned} J\prod_{j=1}^{S-1} B_j &= \Gamma_1^S + \sum_{j=1}^{S-1}\frac{\gamma_j}{1-\gamma_j}\Gamma_j^S \prod_{z=j+1}^{S-1} B_z + \gamma_S \\ &> \Gamma_1^S + \sum_{j=1}^{S-1}\frac{\gamma_j}{1-\gamma_j}\Gamma_j^S + \gamma_S = 1. \end{aligned}$$

of policy. If, however, the product in the denominator is sufficiently less than one, as it may be for firms far downstream, then the second-best subsidy to investments in resilience may be smaller than the first best. Comparing (40) with (35), we see that $\theta_S^\circ > \theta_S^*$; i.e., the government always shaves the second-best subsidy to investments in resilience by final producers relative to the first best; for these firms, there are no downstream distortions, but the markups upstream boost their overall profitability, which tends to lead them to overinvest in resilience compared to the incentives they see in the first best.

Turning to the second best policies that target investments in network thickness, we find once again that the second-best policies mirror their counterparts for protective capabilities; i.e., $\vartheta_s^\circ = \theta_s^\circ$ for all $s \in \{1, 2, \ldots, S\}$. The explanation is the same as before: firms are too small to use their network formation to manipulate their bargaining outcomes and the CES specification of technologies link the consumer-surplus externalities and the profit-stealing externalities generated by investment.

We summarize our findings about the second-best policies in

Proposition 4 *For all $s \geq 1$, the second-best policies for link formation are equal to those for investments in protective capabilities. To achieve the second best, the government subsidizes investments in the most upstream tier. If $\beta_{s+1} \leq \beta_s$ for all $s \in \{0, 1, \ldots, S-1\}$, the second-best subsidies decline with s, and may require a tax for the most downstream tiers. The second-best subsidies may be larger or smaller than their first-best counterparts for $s < S$, but the second-best subsidies (taxes) for investments by final producers are always smaller (larger) than the first-best subsidies (taxes).*

5 Concluding Remarks

We have identified several sources of inefficiency in the market equilibrium of an economy with vertical supply chains and endogenous determination of firms' resilience to supply disturbances. First, in the absence of government policy, firms in adjacent tiers of the supply chain will not choose the socially-optimal volume of input sales. Instead, they will negotiate a contract that calls for more limited sales, in anticipation that the supplier will face a marked-up cost of its own inputs when it subsequently bargains with its own suppliers. The wedge between the private and social incentives for input transactions dictates an optimal subsidy on input sales in all transactions other than between the firms that are most upstream. Second, firms in every tier will not on their own choose the socially-optimal investments to avoid their own supply disturbances. On the one hand, these investments tend to be socially insufficient because firms do not take account that their survival affects the profitability of their downstream customers. On the other hand, these investments may be socially excessive, if the optimal subsidy for sales creates a large profit boost that comes at the expense of the public finances. If the bargaining weights and the labor shares are similar across input tiers but goods become less substitutable as we move down the supply chain, then the optimal subsidies for investments in protective capabilities will be largest upstream and decline monotonically, possibly turning to an optimal tax at some point in the chain. Neither the optimal subsidies on sales nor the optimal subsidies for investments in protective capabilities depend on the number of backward links formed by suppliers, and thus the same subsidies apply for arbitrary networks. Finally, we find a wedge between private and social incentives for firms to form thick supply networks as a hedge against disturbances that might befall their suppliers. As with investments in protective capabilities, firms do not take account that their relationships generate surplus for downstream partners. When firms are too small to use the number of their relationships to manipulate their bargaining position vis-à-vis their suppliers and customers, the optimal policy toward network formation coincides with the optimal policy to promote or discourage investments in protective capabilities.

Political realities may limit the scope for subsidies to firm-to-firm transactions. If so, the government's choice of whether and how to promote resilience takes on a second-best flavor. We considered optimal policies for investments in protective capabilities and for the formation of supplier relationships when a government lacks the ability to use subsidies to counteract the distortionary effects of negotiated input payments. In this setting, optimal policies reflect markups and input shares in all transactions downstream from a targeted tier. Survival and supplier relationships are more socially valuable at upstream stages than at downstream stages due to the cumulative effects of double marginalization. If bargaining weights and production parameters are common across tiers, then the second-best subsidies for investments in protective capabilities and in supplier relationships are larger for producers further upstream. This contrasts with the first-best subsidies, which are constant along the interior of the supply chain when bargaining weights and production parameters are common to all tiers.

We have modeled vertical supply chains in a stylized but realistic way that captures many of the features described in the more descriptive literature. Each firm has multiple suppliers and multiple customers. Bargaining happens sequentially, beginning with final producers that purchase intermediate goods to use in their production processes and proceeding upstream to suppliers that seek inputs to fulfill their procurement contracts. Our bilateral negotiations involve a single buyer and a single seller, not grand coalitions of producers at various stages. Firms form their networks of potential suppliers by investing in bilateral relationships. Resilience reflects deliberate investment. Yet, as with all models of firm-to-firm dealings, the details matter and we recognize that a variety of alternative assumptions may be worthy of further consideration.

First, we have assumed a particular timing and a particular form of contracts. In our model, bargaining between upstream and downstream firms takes place *after* the realization of the supply shocks and firms negotiate only with partners that escape these disturbances. If negotiations were to occur *before* any disruptions, this would open a role for contingent contracts. Payments might be contingent on contract fulfillment, with penalties for failure to deliver. Payments might also be contingent on the size of an upstream firm's investment in resilience (which must be observable if they can be the target of subsidies). Even more sophisticated contracts might allow payments contingent on the resilience of a supplier's own upstream suppliers, or on a firm's realized production costs. Richer contracts would allow firms to mitigate the inefficiencies of double marginalization and to internalize to some extent the externalities that their resilience confers on downstream customers. However, complex contracts that allow for payments based on decisions throughout the network might be needed to achieve full efficiency, especially in a second-best setting in which the government cannot subsidize firm-to-firm transactions. So, the externalities that we highlight would likely still be relevant even in a world with a wider menu of contracts.

Second, if downstream firms could observe investments in protective capabilities before they form their supply networks, they might seek out partners that are more likely to deliver. This would give upstream firms greater incentive to make such investments, thereby mitigating the externality associated with shared benefits. Even if firms could not observe investments before creating their supply chains, they might infer something about such investments if potential suppliers differed in some observable primitives that would affect their incentives to invest.

Finally, our model currently features only idiosyncratic supply shocks and only one place of production. An obvious extension would be to consider correlated shocks, based for example on geography. These would seem particularly important if combined with an extension to *global* supply chains; see, e.g., Grossman et al. (2023) for an analysis of country-wide shocks to input supplies in a two-country model, albeit one with only two tiers of production. The presence of correlated shocks would interact with the possibilities for contract contingencies, as penalties for breach might differ for failures that are specific to a firm versus those that result from more widespread disturbances that are outside a single firm's control. Analyzing optimal unilateral policy and optimal cooperative policy toward resilience in global supply chains will require that cross-country differences in wages, production technologies, and risks of disturbances be taken into account. We regard the modeling of global supply chains with endogenous networks and resilience as an important direction for future research.

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Appendix

This appendix provides proofs and technical details for the statements in the main text. While the main text focuses on the case of homogeneous firms in every tier, with possible differences across tiers, the proofs and arguments in this appendix are developed for the more general case in which within a tier firms differ in Hicks-neutral productivity levels. In particular, it deals with the case in which all firms within a tier are similar ex-ante, when they choose r_s and η_s, $s = 0, 1, ..., S$ (except that firms in tier 0 do not choose η_0 because they have no suppliers). In the next stage the uncertainty is resolve, and a tier-s firm that survives, which happens with probability $1 - \phi_s(r_s)$, discovers its productivity level z.

The density of the productivity distribution in tier s is $f_s(z)$ for $z \geq 0$, so that $\int_0^\infty f_s(z)\,dz = 1$. For an arbitrary function $F(z)$, let

$$\mathbb{E}_s[F(z)] := \int_0^\infty F(z) f_s(z)\,dz$$

be the expected value of $F(z)$ using the tier-s distribution of z. Using this definition, we assume that $\mathbb{E}_s\left[z^{\sigma_{s+1}-1}\right] < \infty$ for $s \in \{0, 1, ..., S-1\}$ and that $\mathbb{E}_S\left[z^{\varepsilon-1}\right] < \infty$. These assumptions are obviously satisfied for distributions with finite supports, but they may require parameter restrictions for distributions with unbounded supports. If, for example, the productivity distributions are Pareto with scale parameter 1 and shape parameter ζ_s in tier s, these assumptions amount to assuming $\zeta_s > \sigma_{s+1} - 1$ for $s \in \{0, 1, ..., S-1\}$ and $\zeta_S > \varepsilon - 1$. In the homogeneous case, which is the focus of the main text, we assume $z = 1$ for every firm in every tier. Therefore in this case $\mathbb{E}_s\left[z^{\sigma_{s+1}-1}\right] = \mathbb{E}_S\left[z^{\varepsilon-1}\right] = 1$.

A tier-s firm with productivity z, $s > 0$, has the production function

$$x_s(z) = z l_s(z)^{\gamma_s} \left(\int_0^{n_s^u} m_{s-1}(i)^{\alpha_s}\,di\right)^{\frac{1-\gamma_s}{\alpha_s}}, \tag{A.1}$$

where $x_s(z)$ is its output, $l_s(z)$ is its employment of labor, $m_{s-1}(i)$ is the amount of the intermediate inputs it obtains from supplier i in tier $s-1$ and n_s^u is the measure (number) of suppliers this firm has in tier $s-1$ (and every supplier delivers a distinct product).[30] And a firm in tier 0 with productivity z has the production function

$$x_0(z) = z l_0(z). \tag{A.2}$$

Note that this production function can be represented by (A.1) with $\gamma_0 = 1$. We will occasionally use this representation.

[30] In order to save on notation, we do not write n_s^u as a function of z. As we show later, every firm in tier s chooses the same fraction of suppliers η_s in the first stage of the game and therefore all tier-s firms have the same number of suppliers.

As we develop the arguments in this appendix, we will point our how various equations look in the homogeneous case. Section A1 describes solutions to the bargaining games between buyers and sellers in all adjacent tiers. Equilibrium outcomes are derived in Section A2. Section A3 characterizes the optimal allocation. We then derive first-best policies in Section A4 and second-best policies in Section A5. Finally, we derive the markup factor in Section **??**.

A1 Bargaining

Bargaining takes place between surviving firms after the resolution of uncertainty. Consider a firm with productivity z in tier s that bargains with one of its suppliers from tier $s-1$. As we will see, in the bargaining with a supplier the firm reaches an agreement that depends on the productivity of the supplier, and there will be no variation in the amount of intermediate inputs it purchases from suppliers with the same productivity. Therefore in equilibrium

$$x_s(z) = z l_s(z)^{\gamma_s} \left(\int_0^\infty n_s^u(z,z') m_{s-1}\left(z,z'\right)^{\alpha_s} dz' \right)^{\frac{1-\gamma_s}{\alpha_s}},$$

where $n_s^u(z,z')$ is the measure (number) of the firm's suppliers in tier $s-1$ with productivity z' and $m_{s-1}(z,z')$ is the amount of the intermediate inputs it buys from each one of them. Because all firms choose the same number n_s^u of upstream suppliers in the first stage of the game,

$$n_s^u(z,z') = n_s^u f_{s-1}(z').$$

Next, let $M_s(z)$ denote the total quantity of output that a tier s firm with productivity z has committed to supply to its tier $s+1$ buyers. Then

$$M_s(z) := \int_0^{n_s^d} m_s^d(i) di, \tag{A.3}$$

where $m_s^d(i)$ is its commitment to customer i and n_s^d is the number of customers of the firm. The CES index of intermediate inputs used by a firm in tier s with productivity z is defined as

$$U_s(z) := \left[\int_0^{n_s^u} m_{s-1}^u(i)^{\alpha_s} di \right]^{\frac{1}{\alpha_s}}, \tag{A.4}$$

where $m_{s-1}^u(i)$ is the quantity it purchases from supplier i. In an equilibrium with a common outcome across firms that bear similar productivity levels, these become

$$M_s(z) := n_s^d \int_1^\infty m_s(z',z) f_{s+1}(z') dz' \tag{A.5}$$

and

$$U_s(z) = n_s^u \left[\int_0^\infty m_{s-1}(z,z')^{\alpha_s} f_{s-1}(z') dz' \right]^{\frac{1}{\alpha_s}}. \tag{A.6}$$

We also define:

$$H_s := \mathbb{E}_s\left[z^{\sigma_{s+1}-1}\right] \text{ for } s \in \{0, 1, ..., S-1\} \tag{A.7}$$

and

$$H_S := \mathbb{E}_S\left[z^{\varepsilon-1}\right]. \tag{A.8}$$

In the symmetric case, where all firms have the same productivity $z = 1$, $H_s = 1$ for $s = 0, 1, ..., S$.

In the following subsections, we solve the sequential Nash-in-Nash bargaining games.

A1.1 Bargaining Between a Tier 0 Firm and Tier 1 Firm

First consider the bargaining problem between a firm in tier 1 with productivity z and a firm in tier 0 with productivity q. The objective is to solve for $m_0(z,q)$ and $t_0(z,q)$ for arbitrary z and q, where $m_0(z,q)$ denotes the quantity sold by a firm in tier 0 with productivity q to a firm in tier 1 with productivity z, and $t_0(z,q)$ denotes the negotiated payment (transfer) by the buyer to the seller. To solve this bargaining problem, note that, using $x_1(z) = M_1(z)$, the total labor hired by a firm with productivity z in tier 1 is given by:

$$l_1(z) = \left[\frac{M_1(z)}{z}\right]^{\frac{1}{\gamma_1}} \left(\int_0^{n_1^u} m_0(i)^{\alpha_s}\, di\right)^{\frac{\gamma_1-1}{\alpha_1\gamma_1}} = \left[\frac{M_1(z)}{z}\right]^{\frac{1}{\gamma_1}} U_1(z)^{\frac{\gamma_1-1}{\gamma_1}}.$$

Now suppose that supplier $i = n_1^u$ has productivity q; i.e., the last supplier in the ordering of $i \in [0, n_1^u]$. Then labor savings from negotiating with this supplier in tier 0 is given by:

$$\begin{aligned}\frac{\partial l_1(z)}{\partial n_1^u} &= -\frac{1-\gamma_1}{\alpha_1\gamma_1}\left[\frac{M_1(z)}{z}\right]^{\frac{1}{\gamma_1}} \left(\int_0^{n_1^u} m_0(i)^{\alpha_1}\, di\right)^{\frac{\gamma_1-1}{\alpha_1\gamma_1}-1} m_0(n_1^u)^{\alpha_1} \\ &= -\frac{1-\gamma_1}{\alpha_1\gamma_1}\frac{l_1(z)m_0(n_1^u)^{\alpha_1}}{\int_0^{n_1^u(z)} m_0(i)^{\alpha_s}\, di} \\ &= -\frac{1-\gamma_1}{\alpha_1\gamma_1} l_1(z)^{\frac{1-\gamma_1(1-\alpha_1)}{1-\gamma_1}} \left[\frac{M_1(z)}{z}\right]^{-\frac{\alpha_1}{1-\gamma_1}} m_0(n_1^u)^{\alpha_1}\end{aligned} \tag{A.9}$$

where the last equality follows from:

$$\int_0^{n_1^u} m_0(i)^{\alpha_1}\, di = l_1(z)^{-\frac{\alpha_1\gamma_1}{1-\gamma_1}} \left[\frac{M_1(z)}{z}\right]^{\frac{\alpha_1}{1-\gamma_1}}.$$

Therefore, when negotiating with a tier 0 supplier with productivity q, the payoff of a tier 1 buyer with productivity z, net of its outside option, is

$$\psi_1^d\left[\tilde{m}_0(z,q), \tilde{t}_0(z,q); z\right] := \frac{1-\gamma_1}{\alpha_1\gamma_1} l_1(z)^{\frac{1-\gamma_1(1-\alpha_1)}{1-\gamma_1}} \left[\frac{M_1(z)}{z}\right]^{-\frac{\alpha_1}{1-\gamma_1}} \tilde{m}_0(z,q)^{\alpha_1} - \tau_0\tilde{t}_0(z,q),$$

where $\tilde{m}_0(z,q) := m_0(n_1^u)$ represents the quantity of sales of supplier $i = n_1^u$, τ_0 is the fraction of the cost of these inputs that the buyer bears in view of the government's policy,[31] and $\tilde{t}_0(z,q)$ is the payment of the buyer to the supplier.

The net payoff of the tier 0 supplier with productivity q is

$$\psi_0^u\left[\tilde{m}_0(z,q),\tilde{t}_0(z,q);q\right] := \tilde{t}_0(z,q) - \frac{\tilde{m}_0(z,q)}{q},$$

and the solution to the bargaining game is obtained from

$$\max_{\tilde{m}_0(z,q),\tilde{t}_0(z,q)} \beta_1 \log\psi_1^d\left[\tilde{m}_0(z,q),\tilde{t}_0(z,q);z\right] + (1-\beta_1)\log\psi_0^u\left[\tilde{m}_0(z,q),\tilde{t}_0(z,q);q\right]. \tag{A.10}$$

The first-order conditions of this problem yield

$$\beta_1\frac{\partial\psi_1^d}{\partial\tilde{m}_0(z,q)}\frac{1}{\psi_1^d} + (1-\beta_1)\frac{\partial\psi_0^u}{\partial\tilde{m}_0(z,q)}\frac{1}{\psi_0^u} = 0,$$
$$-\beta_1\frac{\tau_0}{\psi_1^d} + (1-\beta_1)\frac{1}{\psi_0^u} = 0.$$

Together, they yield the following equation for $\tilde{m}_0(z,q)$:

$$\frac{1-\gamma_1}{\tau_0\gamma_1} l_1(z)^{\frac{1-\gamma_1(1-\alpha_1)}{1-\gamma_1}}\left[\frac{M_1(z)}{z}\right]^{-\frac{\alpha_1}{1-\gamma_1}}\tilde{m}_0(z,q)^{\alpha_1-1} = \frac{1}{q}.$$

We denote this quantity as $m_0(z,q) = \tilde{m}_0(z,q)$, where

$$m_0(z,q) = \left(\frac{1-\gamma_1}{\tau_0\gamma_1}\right)^{\frac{1}{1-\alpha_1}} l_1(z)^{\frac{1-\gamma_1(1-\alpha_1)}{(1-\gamma_1)(1-\alpha_1)}}\left[\frac{M_1(z)}{z}\right]^{-\frac{\alpha_1}{(1-\gamma_1)(1-\alpha_1)}} q^{\sigma_1}. \tag{A.11}$$

Next note that labor employment can be expressed as

$$l_1(z) = \left[\frac{M_1(z)}{z}\right]^{\frac{1}{\gamma_1}} U_1(z)^{\frac{\gamma_1-1}{\gamma_1}}.$$

But given the solution to the input quantities (A.11), the CES aggregator (A.6) for $s = 0$ becomes

$$U_1(z) = n_s^u\left(\frac{1-\gamma_1}{\tau_0\gamma_1}\right)^{\frac{1}{1-\alpha_1}} l_1(z)^{\frac{1-\gamma_1(1-\alpha_1)}{(1-\gamma_1)(1-\alpha_1)}}\left[\frac{M_1(z)}{z}\right]^{-\frac{\alpha_1}{(1-\gamma_1)(1-\alpha_1)}} H_0^{\frac{1}{\alpha_1}}.$$

We therefore obtain

$$l_1(z) = \left(\frac{\tau_0\gamma_1}{1-\gamma_1}\right)^{1-\gamma_1} H_0^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} (n_1^u)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} \frac{M_1(z)}{z}. \tag{A.12}$$

[31] Recall that the government may subsidize such purchases (if $\tau_0 < 1$) or tax them (if $\tau_0 > 1$).

Using this employment level together with (A.11) yields:

$$m_0(z,q) = \tilde{C}_0 \left(n_1^u\right)^{\frac{(1-\alpha_1)\gamma_1-1}{\alpha_1}} \frac{M_1(z)}{z} q^{\sigma_1}. \tag{A.13}$$

where $\sigma_1 = 1/\left(1-\alpha_1\right)$ and

$$\tilde{C}_0 = \left(\frac{1-\gamma_1}{\tau_0\gamma_1}\right)^{\gamma_1} H_0^{\frac{(1-\alpha_1)\gamma_1-1}{\alpha_1}} \tag{A.14}$$

As is evident from (A.13), the buyer with productivity z reaches the same agreement on purchases of intermediate inputs with every supplier whose productivity is q. In the symmetric case $z = q = 1$ for all firms, we have $H_0 = 1$ and $M_1 = n_1^d m_1$, and therefore

$$m_0 = \left(\frac{1-\gamma_1}{\tau_0\gamma_1}\right)^{\gamma_1} \left(n_1^u\right)^{\frac{(1-\alpha_1)\gamma_1-1}{\alpha_1}} n_1^d m_1.$$

This is a recursive equation, stating how m_0 depends on m_1, which is used in the main text.

We next solve for the transfer. The first-order condition with respect to $\tilde{t}_0$ in problem (A.10), together with the definitions of the net payoffs ψ_0^u and ψ_1^d, yield

$$\begin{aligned} & \beta_1\tau_0\left[\tilde{t}_0(z,q) - \frac{m_0(z,q)}{q}\right] \\ = \quad & (1-\beta_1)\left\{\frac{1-\gamma_1}{\alpha_1\gamma_1} l_1(z)^{\frac{1-\gamma_1(1-\alpha_1)}{1-\gamma_1}} \left[\frac{M_1(z)}{z}\right]^{-\frac{\alpha_1}{1-\gamma_1}} m_0(z,q)^{\alpha_1} - \tau_0\tilde{t}(z,q)\right\}. \end{aligned}$$

It is evident from this equation that the solution to the transfer is the same for every supplier with productivity q. We denote this amount as $t_0(z,q) = \tilde{t}_0(z,q)$. Combining this equation with (A.12) and (A.13), we then obtain

$$\begin{aligned} \tau_0 t_0(z,q) &= (1-\beta_1)\left[\frac{\tau_0}{\alpha_1}\left(n_1^u\right)^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}} \left(H_0\right)^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}} \left(\frac{1-\gamma_1}{\tau_0\gamma_1}\right)^{\gamma_1} q^{\sigma_1}\frac{M_1(z)}{zq}\right] + \beta_1\tau_0\frac{m_0(z,q)}{q} \\ &= (1-\beta_1)\left[\frac{\tau_0}{\alpha_1}\frac{m_0(z,q)}{q}\right] + \beta_1\tau_0\frac{m_0(z,q)}{q} \\ &= \tau_0\frac{m_0(z,q)}{q}\left[(1-\beta_1)\frac{1}{\alpha_1} + \beta_1\right], \end{aligned}$$

or

$$t_0(z,q) = \frac{m_0(z,q)}{q}\left[\beta_1 + (1-\beta_1)\frac{\sigma_1}{\sigma_1-1}\right] = \frac{m_0(z,q)}{q}\mu_0, \tag{A.15}$$

where

$$\mu_0 := \beta_1 + (1-\beta_1)\frac{\sigma_1}{\sigma_1-1}.$$

For future reference note that, using (A.15), the equilibrium net payoff of a supplier in tier 0 with productivity q from bargaining with a buyer from tier 1 with productivity z, is

$$\psi_0^u(z,q) = t_0(z,q) - \frac{m_0(z,q)}{q} = \frac{m_0(z,q)}{q}\frac{1-\beta_1}{\sigma_1 - 1},$$

and, using (A.13) and (A.15), aggregate payments of the buyer from tier 1 with productivity z for its tier 0 inputs are

$$T_1(z) := \tau_0\mu_0 n_1^u \int_0^\infty \frac{m_0(z,q)}{q} f_0(q)dq = \tau_0\mu_0 H_0 \tilde{C}_0 \left(n_1^u\right)^{-\frac{(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} \frac{M_1(z)}{z}. \tag{A.16}$$

A1.1.1 Deviant Outcomes

If a tier-1 firm with productivity z were to invest in network thickness. an amount that differs from the equilibrium level, the fraction of suppliers in tier 0 with whom it would have links would be $\tilde{\eta}_1$, different from the equilibrium fraction η_1 that is chosen in the first stage of the game, i.e., $\tilde{\eta}_1 \neq \eta_1$. In this event the formulas from subsection A1.1 still apply, as long as n_1^u is interpreted to be the number of suppliers actually available to this buyer. Alternatively, one can replace n_1^u with $\tilde{n}_1^u$, where the latter is interpreted to be the number of suppliers available to a tier-1 firm that forms $\tilde{\eta}_1 N_0$ links in the first stage of the game. In this case (A.13) implies the transacted quantity

$$\tilde{m}_0(z,q) = \tilde{C}_0 \left(\tilde{n}_1^u\right)^{\frac{(1-\alpha_1)\gamma_1 - 1}{\alpha_1}} \frac{M_1(z)}{z} q^{\sigma_1}. \tag{A.17}$$

A1.2 Bargaining Between a Tier 1 Firm and a Tier 2 Firm

We now solve the bargaining problem between firms in the interior of the supply chain. Consider negotiations between a firm in tier 2 with productivity z and a firm in tier 1 with productivity q. In this bargaining game, the tier-2 firm saves labor costs when purchasing inputs from the tier-1 firm. Using the same arguments as in the previous section, we obtain a formula similar to (A.9):

$$\frac{\partial l_2(z)}{\partial n_2^u} = -\frac{1-\gamma_2}{\alpha_2\gamma_2} l_2(z)^{\frac{1-\gamma_2(1-\alpha_2)}{1-\gamma_2}} \left[\frac{M_2(z)}{z}\right]^{-\frac{\alpha_2}{1-\gamma_2}} m_1 \left(n_2^u\right)^{\alpha_2}$$

Therefore the net payoff of the tier-2 firm (net of its outside option), is

$$\psi_2^d\left[\tilde{m}_1(z,q), \tilde{t}_1(z,q); z\right] := \frac{1-\gamma_2}{\alpha_2\gamma_2} l_2(z)^{\frac{1-\gamma_2(1-\alpha_2)}{1-\gamma_2}} \left[\frac{M_2(z)}{z}\right]^{-\frac{\alpha_2}{1-\gamma_2}} \tilde{m}_1(z,q)^{\alpha_2} - \tau_1\tilde{t}_1(z,q).$$

Calculation of the net payoff of a supplier in tier 1 with productivity q is somewhat more subtle, because a breakdown in negotiations between a tier-2 firm and a tier-1 firm will affect the negotiations that the tier-1 firm subsequently has with its tier-0 suppliers. On the equilibrium path, this firm is committed to supplying $M_1(q)$ units to its downstream buyers. Using (A.3) and (A.12), employment by this supplier is

$$
\begin{aligned}
l_1(q) &= \left(\frac{\tau_0\gamma_1}{1-\gamma_1}\right)^{1-\gamma_1} (H_0)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} (n_1^u)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} \frac{M_1(q)}{q} \\
&= \left(\frac{\tau_0\gamma_1}{1-\gamma_1}\right)^{1-\gamma_1} (H_0)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} (n_1^u)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} \frac{1}{q}\int_0^{n_1^d} m_1^d(i)di.
\end{aligned}
$$

Therefore the extra labor cost of supplying $\tilde{m}_1(z,q) = m_1^d(n_1^d)$ units of the intermediate input to a buyer with productivity z is

$$
\frac{\partial l_1(q)}{\partial n_1^d} = \left(\frac{\tau_0\gamma_1}{1-\gamma_1}\right)^{1-\gamma_1} (H_0)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} (n_1^u)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} \frac{1}{q}\tilde{m}_1(z,q).
$$

But the tier-1 firm also anticipates its transfer bill $T_1(q)$ to change if it drops a buyer who purchases $\tilde{m}_1(z,q)$ units of the intermediate input, because the negotiation outcomes with its own suppliers depend on $M_1(q)$. Using (A.3) and (A.16), we obtain these savings to be

$$
\begin{aligned}
\frac{\partial T_1(q)}{\partial n_1^d} &= \tau_0\mu_0 H_0 \tilde{C}_0 (n_1^u)^{-\frac{(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} \frac{\tilde{m}_1(z,q)}{q} \\
&= \tau_0\mu_0 \left(\frac{1-\gamma_1}{\tau_0\gamma_1}\right)^{\gamma_1} H_0^{-\frac{(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} (n_1^u)^{-\frac{(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} \frac{\tilde{m}_1(z,q)}{q}.
\end{aligned}
$$

Subgame perfection requires that we take account of this change in the aggregate payments along the (off-equilibrium) path of a breakdown in negotiations. It follows that the net payoff of the tier-1 supplier with productivity q is

$$
\begin{aligned}
\psi_1^u\left[\tilde{m}_1(z,q), \tilde{t}_1(z,q); q\right] \quad &: \quad = \tilde{t}_1(z,q) - \frac{\partial l_1(q)}{\partial n_1^d} - \frac{\partial T_1(q)}{\partial n_1^d} \\
&= \tilde{t}_1(z,q) - c_1\frac{\tilde{m}_1(z,q)}{q}.
\end{aligned}
$$

where

$$
\begin{aligned}
c_1 &:= (n_1^u)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} H_0^{-\frac{(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} \tau_0 \left(\frac{1-\gamma_1}{\tau_0\gamma_1}\right)^{\gamma_1} \frac{1}{1-\gamma_1} B_1 \qquad \text{(A.18)} \\
&= (n_1^u)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} H_0^{-\frac{1-\gamma_1}{\alpha_1\gamma_1}} \tilde{C}_0^{\frac{\gamma_1-1}{\gamma_1}} \frac{1}{\gamma_1} B_1
\end{aligned}
$$

and

$$
B_1 := \mu_0(1-\gamma_1) + \gamma_1.
$$

The coefficient c_1/q represents the cost of producing a marginal unit of $\tilde{m}_1(z,q)$ by a firm with productivity q. In the homogenous case $q = H_0 = 1$, and in the absence of policies $\tau_0 = 1$. In this case

$$c_1 := (n_1^u)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} \left(\frac{1-\gamma_1}{\gamma_1}\right)^{\gamma_1} \frac{1}{1-\gamma_1} B_1, \tag{A.19}$$

which is the expression used in the main text (recalling that $\alpha_1 = (\sigma_1 - 1)/\sigma_1$).

Now that we have the surplus each side achieves from an agreement calling for $\tilde{m}_1(z,q)$ and $\tilde{t}_1(z,q)$, we can derive the Nash bargaining solution

$$\begin{aligned} & [m_1(z,q), t_1(z,q)] \\ = \ & \arg \max_{\tilde{m}_1(z,q), \tilde{t}_1(z,q)} \beta_2 \log \psi_2^d \left[\tilde{m}_1(z,q), \tilde{t}_1(z,q); z\right] + (1-\beta_2) \log \psi_1^u \left[\tilde{m}_1(z,q), \tilde{t}_1(z,q); q\right]. \end{aligned}$$

The first-order conditions of this problem yield

$$\frac{1-\gamma_2}{c_1 \tau_1 \gamma_2} l_2(z)^{\frac{1-\gamma_2(1-\alpha_2)}{1-\gamma_2}} \left[\frac{M_2(z)}{z}\right]^{-\frac{\alpha_2}{1-\gamma_2}} m_1(z,q)^{\alpha_2 - 1} = \frac{1}{q}.$$

We see again that the solution is the same for every supplier with productivity q, and

$$m_1(z,q) = \left(\frac{1-\gamma_2}{c_1 \tau_1 \gamma_2}\right)^{\frac{1}{1-\alpha_2}} l_2(z)^{\frac{1-\gamma_2(1-\alpha_2)}{(1-\gamma_2)(1-\alpha_2)}} \left[\frac{M_2(z)}{z}\right]^{-\frac{\alpha_2}{(1-\alpha_2)(1-\gamma_2)}} q^{\frac{1}{1-\alpha_2}}. \tag{A.20}$$

Note the resemblance between (A.20) and (A.11). Now use

$$l_2(z) = \left(\frac{M_2(z)}{z}\right)^{\frac{1}{\gamma_2}} U_2(z)^{\frac{\gamma_2 - 1}{\gamma_2}}$$

together with the definition of $U_2(z)$ in (A.4) and (A.20), to obtain:

$$l_2(z) = \left(\frac{1-\gamma_2}{c_1 \tau_1 \gamma_2}\right)^{\gamma_2 - 1} (H_1)^{-\frac{(1-\alpha_2)(1-\gamma_2)}{\alpha_2}} (n_2^u)^{-\frac{(1-\alpha_2)(1-\gamma_2)}{\alpha_2}} \frac{M_2(z)}{z}. \tag{A.21}$$

Substituting this expression for $l_2(z)$ into (A.20) then delivers

$$m_1(z,q) = \tilde{C}_1 (n_2^u)^{\frac{(1-\alpha_2)\gamma_2 - 1}{\alpha_2}} \frac{M_2(z)}{z} q^{\sigma_2}, \tag{A.22}$$

where

$$\tilde{C}_1 := \left(\frac{1-\gamma_2}{c_1 \tau_1 \gamma_2}\right)^{\gamma_2} (H_1)^{\frac{(1-\alpha_2)\gamma_2 - 1}{\alpha_2}}.$$

In the symmetric case $z = q = 1$ for all firms, which implies that $H_1 = 1$, $M_2 = n_2^d m_2$ and $\tilde{C}_1$ and c_1 are constants. In this case (A.22) yields the recursive equation

$$m_1 = \tilde{C}_1 (n_2^u)^{\frac{(1-\alpha_2)\gamma_2 - 1}{\alpha_2}} n_2^d m_2$$

that is used in the main text, with $\tau_1 = 1$.

Next, we compute the size of the transfers. The first-order conditions of the bargaining problem, evaluated at $\tilde{t}_1(z,q) = t_1(z,q)$ and $\tilde{m}_1(z,q) = m_1(z,q)$, imply

$$\begin{aligned}&\tau_1\beta_2\left[t_1(z,q) - c_1\frac{m_1(z,q)}{q}\right] \\ = \quad &(1-\beta_2)\left\{\frac{1-\gamma_2}{\alpha_2\gamma_2}l_2(z)^{\frac{1-\gamma_2(1-\alpha_2)}{1-\gamma_2}}\left[\frac{M_2(z)}{z}\right]^{-\frac{\alpha_2}{1-\gamma_2}} m_1(z,q)^{\alpha_1} - \tau_1 t_1(z,q)\right\}.\end{aligned} \tag{A.23}$$

Using (A.21) and (A.22), this yields

$$\tau_1\beta_2\left[t_1(z,q) - c_1\frac{m_1(z,q)}{q}\right] = (1-\beta_2)\left[\frac{\tau_1 c_1}{\alpha_2}\frac{m_1(z,q)}{q} - \tau_1 t_1(z,q)\right]$$

and therefore

$$t_1(z,q) = \left[\beta_2 + (1-\beta_2)\frac{\sigma_2}{\sigma_2 - 1}\right]c_1\frac{m_1(z,q)}{q} = \mu_1 c_1\frac{m_1(z,q)}{q}. \tag{A.24}$$

From this equation we obtain aggregate payments to suppliers by a tier-2 firm with productivity z:

$$T_2(z) = c_1\tau_1\mu_1 H_1\tilde{C}_1(n_2^u)^{-\frac{(1-\gamma_2)(1-\alpha_2)}{\alpha_2}}\frac{M_2(z)}{z}. \tag{A.25}$$

Note the similarity of this equation to (A.16) (where $c_0 = 1$).

A1.2.1 Deviant Outcomes

In this subsection, we derive the bargaining outcome that arises when the upstream firm with productivity q has links with fraction $\tilde{\eta}_1$ of suppliers in tier 0, which is different from the fraction η_1 chosen by the other firms in tier 1. When this occurs, this deviant firm has $\tilde{n}_1^u$ suppliers and the solution to its bargaining game with a buyer with productivity z in tier 2 yields

$$\tilde{m}_1(z,q) = \left(\frac{1-\gamma_2}{\tilde{c}_1\tau_1\gamma_2}\right)^{\frac{1}{1-\alpha_2}} l_2(z)^{\frac{1-\gamma_2(1-\alpha_2)}{(1-\gamma_2)(1-\alpha_2)}}\left[\frac{M_2(z)}{z}\right]^{-\frac{\alpha_2}{(1-\alpha_2)(1-\gamma_2)}} q^{\sigma_2} \tag{A.26}$$

where

$$\tilde{c}_1 = (\tilde{n}_1^u)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}}(H_0)^{-\frac{1-\gamma_1}{\alpha_1\gamma_1}}\tilde{C}_0^{\frac{\gamma_1-1}{\gamma_1}}\frac{1}{\gamma_1}B_1.$$

Equation (A.21) for $l_2(z)$ still holds because all other firms do choose η_1 and have the cost parameter c_1. Therefore

$$\begin{aligned}\tilde{m}_1(z,q) \quad = \quad &\left(\frac{1-\gamma_2}{\tau_1\gamma_2}\right)^{\gamma_2}(\tilde{c}_1)^{-\frac{1}{1-\alpha_2}}(c_1)^{-\frac{\gamma_2(1-\alpha_2)-1}{1-\alpha_2}} \\ &\times(H_1)^{\frac{\gamma_2(1-\alpha_2)-1}{\alpha_2}}(n_2^u)^{\frac{\gamma_2(1-\alpha_2)-1}{\alpha_2}}\frac{M_2(z)}{z}q^{\sigma_2}.\end{aligned} \tag{A.27}$$

The first-order condition (A.23) still applies, except that the transacted quantity $m_1(z,q)$ must be replaced by the quantity in (A.27) and c_1 must be replaced by $\tilde{c}_1$. Making these substitutions, we obtain the deviant's transfer

$$\tilde{t}_1(z,q) = \mu_1 \tilde{c}_1 \frac{\tilde{m}_1(z,q)}{q}.$$

A1.3 Bargaining Between a Buyer in Tier $s < S$ and a Supplier in Tier $s-1$: Generalization

Now consider bargaining between a buyer with productivity z in an arbitrary tier s and a seller with productivity q in tier $s-1$, for $s \in \{2, 3, ..., S-1\}$. Using the arguments for $s = 2$ from the previous section, the surplus of the downstream buyer is

$$\psi_s^d \left[\tilde{m}_{s-1}(z,q), \tilde{t}_{s-1}(z,q); z\right] = \frac{1-\gamma_s}{\alpha_s \gamma_s} l_s(z)^{\frac{1-\gamma_s(1-\alpha_s)}{1-\gamma_s}} \left[\frac{M_s(z)}{z}\right]^{-\frac{\alpha_s}{1-\gamma_s}} \tilde{m}_{s-1}(z,q)^{\alpha_s} - \tau_{s-1}\tilde{t}_{s-1}(z,q),$$

and the surplus of the upstream firm is

$$\psi_{s-1}^u \left[\tilde{m}_{s-1}(z,q), \tilde{t}_{s-1}(z,q); q\right] = \tilde{t}_{s-1}(z,q) - c_{s-1}\frac{\tilde{m}_{s-1}(z,q)}{q}, \tag{A.28}$$

where

$$c_{s-1} := (n_{s-1}^u)^{-\frac{(1-\alpha_{s-1})(1-\gamma_{s-1})}{\alpha_{s-1}}} (H_{s-2})^{-\frac{1-\gamma_{s-1}}{\alpha_{s-1}\gamma_{s-1}}} \tilde{C}_{s-2}^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} \frac{1}{\gamma_{s-1}} B_{s-1} \tag{A.29}$$

and

$$\begin{aligned}
B_{s-1} &:= \mu_{s-2}(1-\gamma_{s-1}) + \gamma_{s-1}, \\
\mu_{s-2} &:= \beta_{s-1} + (1-\beta_{s-1})\frac{\sigma_{s-1}}{\sigma_{s-1}-1}, \\
\tilde{C}_{s-2} &:= \left(\frac{1-\gamma_{s-1}}{c_{s-2}\tau_{s-2}\gamma_{s-1}}\right)^{\gamma_{s-1}} (H_{s-2})^{\frac{(1-\alpha_{s-1})\gamma_{s-1}-1}{\alpha_{s-1}}}.
\end{aligned}$$

The resulting solutions are

$$m_{s-1}(z,q) = \tilde{C}_{s-1}(n_s^u)^{\frac{(1-\alpha_s)\gamma_s-1}{\alpha_s}} \frac{M_s(z)}{z} q^{\sigma_s} \quad \text{for } s \in \{2, ..., S-1\}, \tag{A.30}$$

$$t_{s-1}(z,q) = \mu_{s-1} c_{s-1} \frac{m_{s-1}(z,q)}{q} \quad \text{for } s \in \{2, ..., S-1\}, \tag{A.31}$$

$$l_s(z) = \left(\frac{1-\gamma_s}{c_{s-1}\tau_{s-1}\gamma_s}\right)^{\gamma_s - 1} (H_{s-1})^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}} (n_s^u)^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}} \frac{M_s(z)}{z} \tag{A.32}$$

for $s \in \{1, ..., S-2\}$,

$$T_s(z) = c_{s-1}\tau_{s-1}\mu_{s-1}H_{s-1}\tilde{C}_{s-1}\left(n_s^u\right)^{-\frac{(1-\gamma_s)(1-\alpha_s)}{\alpha_s}}\frac{M_s(z)}{z} \quad \text{for } s \in \{1, ..., S-2\}. \tag{A.33}$$

In the symmetric case, where $z = q = 1$ for all firms, we have $H_s = 1$, $M_s = n_s^d m_s^d$ and $\tilde{C}_{s-1}$ and c_{s-1} are constants. Then (A.30) provides the recursive equation

$$m_{s-1}(z, q) = \tilde{C}_{s-1}(n_s^u)^{\frac{(1-\alpha_s)\gamma_s - 1}{\alpha_s}} n_s^d m_s^d \quad \text{for } s \in \{2, ..., S-1\}$$

that is used in the main text.

A1.3.1 Deviant Outcomes

Following the arguments in section A1.2.1, a deviant supplier with productivity q who formed links with a fraction $\tilde{\eta}_{s-1}$ of firms in the tier above him while all other suppliers in his tier formed links with the fractions η_{s-1} of these firms, reaches an agreement with a buyer with productivity z that yields

$$\begin{aligned} \tilde{m}_{s-1}(z, q) &= \left(\frac{1-\gamma_s}{\tau_{s-1}\gamma_s}\right)^{\gamma_s} (\tilde{c}_{s-1})^{-\frac{1}{1-\alpha_s}} (c_{s-1})^{-\frac{\gamma_s(1-\alpha_s)-1}{1-\alpha_s}} \\ &\quad \times (H_{s-1})^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}} (n_s^u)^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}} \frac{M_s(z)}{z} q^{\sigma_s}, \end{aligned} \tag{A.34}$$

$$\tilde{t}_{s-1} = \mu_{s-1}\tilde{c}_{s-1}\frac{\tilde{m}_{s-1}(z, q)}{q}, \tag{A.35}$$

where

$$\tilde{c}_{s-1} := (\tilde{n}_{s-1}^u)^{-\frac{(1-\alpha_{s-1})(1-\gamma_{s-1})}{\alpha_{s-1}}} (H_{s-2})^{-\frac{1-\gamma_{s-1}}{\alpha_{s-1}\gamma_{s-1}}} \tilde{C}_{s-2}^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} \frac{1}{\gamma_{s-1}} B_{s-1} \tag{A.36}$$

A1.4 Bargaining Between a Tier S Firm and a Tier $S-1$ Firm

A final goods producer with productivity z faces the inverse demand function:

$$p_S(z) = \left[\frac{A}{x_S(z)}\right]^{\frac{1}{\varepsilon}}.$$

Therefore, its operating profits, comprising revenue less production costs, are

$$\pi_S(z) = A^{\frac{1}{\varepsilon}} z^{\frac{\varepsilon-1}{\varepsilon}} l_S(z)^{\frac{\gamma_S(\varepsilon-1)}{\varepsilon}} \left[\int_0^{n_S^u} m_{S-1}(i)^{\alpha_S}\, di\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S \varepsilon}} - l_S(z) - \tau_{S-1}\int_0^{n_S^u} t_{S-1}(i)\, di,$$

where $l_S(z)$ is labor employment, n_S^u is the number of its suppliers, $m_{S-1}(i)$ is the quantity of purchased from supplier i, and $t_{S-1}(i)$ is the payment to supplier i. Choosing $l_S(z)$ to maximize profits yields

$$l_S(z) = A^{\frac{1}{\varepsilon-\gamma_S(\varepsilon-1)}} z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}} \left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-\gamma_S(\varepsilon-1)}} \left[\int_0^{n_S^u} m_{S-1}(i)^{\alpha_S}\, di\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}}, \tag{A.37}$$

and the resulting profits are

$$\pi_S(z) = C_\pi z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}} \left[\int_0^{n_S^u} m_{S-1}(i)^{\alpha_S}\, di\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}} - \tau_{S-1}\int_0^{n_S^u} t_{S-1}(i)\, di, \tag{A.38}$$

where

$$C_\pi := A^{\frac{1}{\varepsilon-\gamma_S(\varepsilon-1)}} \left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\frac{\gamma_S(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}} \frac{\varepsilon-\gamma_S(\varepsilon-1)}{\varepsilon}. \tag{A.39}$$

Now consider bargaining between this final-goods producer and a tier $S-1$ supplier of intermediate inputs with productivity q. For the firm in tier $S-1$ the surplus is similar to what we calculated in the previous section, i.e.,

$$\psi_{S-1}^u\left[\tilde{m}_{S-1}(z,q), \tilde{t}_{S-1}(z,q); q\right] = \tilde{t}_{S-1}(z,q) - c_{S-1}\frac{\tilde{m}_{S-1}(z,q)}{q},$$

where

$$c_{S-1} := \left(n_{S-1}^u\right)^{-\frac{(1-\alpha_{S-1})(1-\gamma_{S-1})}{\alpha_{S-1}}} \left(H_{S-2}\right)^{-\frac{1-\gamma_{S-1}}{\alpha_{S-1}\gamma_{S-1}}} \tilde{C}_{S-2}^{\frac{\gamma_{S-1}-1}{\gamma_{S-1}}} \frac{1}{\gamma_{S-1}} B_{S-1} \tag{A.40}$$

and

$$\begin{aligned}
B_{S-1} &:= \mu_{S-2}(1-\gamma_{S-1}) + \gamma_{S-1}, \\
\mu_{S-2} &:= \beta_{S-1} + (1-\beta_{S-1})\frac{\sigma_{S-1}}{\sigma_{S-1}-1}, \\
\tilde{C}_{S-2} &:= \left(\frac{1-\gamma_{S-1}}{c_{S-2}\tau_{S-2}\gamma_{S-1}}\right)^{\gamma_{S-1}} \left(H_{S-2}\right)^{\frac{(1-\alpha_{S-1})\gamma_{S-1}-1}{\alpha_{S-1}}}.
\end{aligned}$$

For the downstream firm, not reaching an agreement with a seller with productivity q who supplies $\tilde{m}_{S-1}(z,q)$ units of the intermediate input for payment $\tilde{t}_{S-1}(z,q)$ would reduce operating profits by $\partial\pi_S(z)/\partial n_S^u$, evaluated at $m_{S-1}(n_S^u) = \tilde{m}_{S-1}(z,q)$ and $t_{S-1}(n_S^u) = \tilde{t}_{S-1}(z,q)$. Therefore, using (A.38), the buyer's surplus from the relationship is

$$\psi_S^d\left[\tilde{m}_{S-1}(z,q),\tilde{t}_{S-1}(z,q);z\right]=C_\pi^d z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}}U_S\left(z\right)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}-\alpha_S}\tilde{m}_{S-1}(z,q)^{\alpha_S} \\ -\tau_{S-1}\tilde{t}_{S-1}(z,q). \tag{A.41}$$

where

$$C_\pi^d:=A^{\frac{1}{\varepsilon-\gamma_S(\varepsilon-1)}}\left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\frac{\gamma_S(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}}\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S\varepsilon}.$$

The solution to the bargaining game between these two firms is

$$\begin{aligned}&[m_{S-1}(z,q),t_{S-1}(z,q)]\\ =\;&\arg\max_{\tilde{m}_{S-1}(z,q),\tilde{t}_{S-1}(z,q)}\{\beta_S\log\psi_S^d\left[\tilde{m}_{S-1}(z,q),\tilde{t}_{S-1}(z,q);z\right]\\ &+(1-\beta_S)\log\psi_{S-1}^u\left[\tilde{m}_{S-1}(z,q),\tilde{t}_{S-1}(z,q);q\right]\}.\end{aligned}$$

The first-order conditions for this maximization problem are

$$\frac{\beta_S}{\psi_S^d}\frac{\partial\psi_S^d}{\partial\tilde{m}_{S-1}(z,q)}+\frac{1-\beta_S}{\psi_{S-1}^u}\frac{\partial\psi_{S-1}^u}{\partial\tilde{m}_{S-1}(z,q)}=0,$$
$$\frac{-\beta_S\tau_{S-1}}{\psi_S^d}+\frac{1-\beta_S}{\psi_{S-1}^u}=0.$$

They yield

$$q^{\frac{1}{1-\alpha_S}}\left(\frac{C_\pi^d}{c_{S-1}\tau_{S-1}}\right)^{\frac{1}{1-\alpha_S}}\alpha_S^{\frac{1}{1-\alpha_S}}z^{\frac{\varepsilon-1}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}}U_S\left(z\right)^{\frac{(1-\gamma_S)(\varepsilon-1)}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}-\frac{\alpha_S}{1-\alpha_S}} \\ =m_{S-1}(z,q), \tag{A.42}$$

and the solution

$$\begin{aligned}m_{S-1}(z,q)\;&=\;q^{\frac{1}{1-\alpha_S}}\left(\frac{C_\pi^d}{c_{S-1}\tau_{S-1}}\right)^{\frac{1}{1-\alpha_S}}\alpha_S^{\frac{1}{1-\alpha_S}}\\ &\quad\times z^{\frac{\varepsilon-1}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}}U_S(z)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)-\alpha_S}{(1-\alpha_S)(\varepsilon-1)(1-\gamma_S)+(1-\alpha_S)}}.\end{aligned} \tag{A.43}$$

Raising both sides of this equation to the power α_S, multiplying by $n_S^u f_{S-1}\left(q\right)$ and integrating over q, provides a solution for the CES index

$$U_S(z)=H_{S-1}^{\frac{[\varepsilon-\gamma_S(\varepsilon-1)](1-\alpha_S)}{\alpha_S}}\left(\alpha_S\frac{C_\pi^d}{c_{S-1}\tau_{S-1}}\right)^{\varepsilon-\gamma_S(\varepsilon-1)}z^{\varepsilon-1}(n_S^u)^{\frac{[\varepsilon-\gamma_S(\varepsilon-1)](1-\alpha_S)}{\alpha_S}} \tag{A.44}$$

Substituting (A.44) into (A.43) then yields[32]

$$m_{S-1}(z,q) = C_{S-1}A(n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} q^{\sigma_S} z^{\varepsilon-1}, \tag{A.45}$$

where

$$\begin{aligned} C_{S-1} \quad : &= (H_{S-1})^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} \left(\frac{1}{c_{S-1}\tau_{S-1}}\right)^{\varepsilon-\gamma_S(\varepsilon-1)} \\ &\times \left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S(\varepsilon-1)} \left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{\varepsilon}. \end{aligned} \tag{A.46}$$

We now solve for $t_{S-1}(z,q)$. From the first-order condition of the maximization problem we have

$$\begin{aligned} &\beta_S\tau_{S-1}\left[t_{S-1}(z,q) - c_{S-1}\frac{m_{S-1}(z,q)}{q}\right] \\ &= (1-\beta_S)\left[C_\pi^d z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}} U_S(z)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}-\alpha_S} m_{S-1}(z,q)^{\alpha_S} - \tau_{S-1}t_{S-1}(z,q)\right] \end{aligned}$$

Using (A.44) and (A.45) we obtain

$$\begin{aligned} &C_\pi^d z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}} U_S(z)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}-\alpha_S} m_{S-1}(z,q)^{\alpha_S} \\ &= \frac{1}{\alpha_S} A z^{\varepsilon-1}(n_S^u)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}-1} (H_{S-1})^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}-1} \left(\frac{1}{c_{S-1}\tau_{S-1}}\right)^{\varepsilon-\gamma_S(\varepsilon-1)} c_{S-1}\tau_{S-1} \\ &\times \left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S(\varepsilon-1)} \left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{\varepsilon} q^{\sigma_S-1} \\ &= c_{S-1}\frac{m_{S-1}(z,q)}{q}\tau_{S-1}\frac{1}{\alpha_S}. \end{aligned}$$

Therefore

$$t_{S-1}(z,q) = c_{S-1}\frac{m_{S-1}(z,q)}{q}\left[\beta_S + (1-\beta_S)\frac{\sigma_S}{\sigma_S-1}\right] = c_{S-1}\mu_{S-1}\frac{m_{S-1}(z,q)}{q} \tag{A.47}$$

and

[32] In the symmetric case, where $z = q = 1$, this equation yields

$$m_{S-1} = C_{S-1}A(n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1},$$

where C_S is a constant.

$$T_S(z) = Ac_{S-1}\tau_{S-1}\mu_{S-1}C_{S-1}H_{S-1}(n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}} z^{\varepsilon-1}. \tag{A.48}$$

This completes the solutions of the bargaining games in all tiers.

A1.4.1 Deviant Outcomes

Consider again a deviant firm in tier $S-1$ with productivity q that has chosen $\tilde{\eta}_{S-1}$ while all other firms in its tier have chosen η_{S-1}. For such a firm condition (A.42) is satisfied, except that c_{S-1} and $m_{S-1}(z,q)$ have to be replaced with $\tilde{c}_{S-1}$ and $\tilde{m}_{S-1}(z,q)$

$$q^{\frac{1}{1-\alpha_S}} \left(\frac{C_\pi^d}{\tilde{c}_{S-1}\tau_{S-1}}\right)^{\frac{1}{1-\alpha_S}} \alpha_S^{\frac{1}{1-\alpha_S}} z^{\frac{\varepsilon-1}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}} U_S(z)^{\frac{(1-\gamma_S)(\varepsilon-1)}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}-\frac{\alpha_S}{1-\alpha_S}} \tag{A.49}$$
$$= \tilde{m}_{S-1}(z,q),$$

where

$$\tilde{c}_{S-1} := (\tilde{n}_{S-1}^u)^{-\frac{(1-\alpha_{S-1})(1-\gamma_{S-1})}{\alpha_{S-1}}} (H_{S-2})^{-\frac{1-\gamma_{S-1}}{\alpha_{S-1}\gamma_{S-1}}} \tilde{C}_{S-2}^{\frac{\gamma_{S-1}-1}{\gamma_{S-1}}} \frac{1}{\gamma_{S-1}} B_{S-1}.$$

The formula for $\tilde{c}_{S-1}$ is the same as for c_{S-1}, except that $\tilde{n}_{S-1}^u$ replaces n_{S-1}^u, i.e., the actual number of suppliers for the deviant firm is used in the definition of $\tilde{c}_{S-1}$. Combining with (A.45), we therefore obtain:

$$\tilde{m}_{S-1}(z,q) = \left(\frac{c_{S-1}}{\tilde{c}_{S-1}}\right)^{\frac{1}{1-\alpha_S}} m_{S-1}(z,q),$$

or

$$\tilde{m}_{S-1}(z,q) = \left(\frac{c_{S-1}}{\tilde{c}_{S-1}}\right)^{\frac{1}{1-\alpha_S}} C_{S-1}A(n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} q^{\sigma_S}(z)^{\varepsilon-1}. \tag{A.50}$$

In addition, the first-order condition of the maximization problem with respect to the transfer still holds, using $\tilde{m}_{S-1}(z,q)$ and $\tilde{c}_{S-1}$ instead of $m_{S-1}(z,q)$ and c_{S-1} . This yields the transfer:

$$\tilde{t}_{S-1}(z,q) = \mu_{S-1}\tilde{c}_{S-1}\frac{\tilde{m}_{S-1}(z,q)}{q}. \tag{A.51}$$

We have derived in this section solutions to the bargaining games in all tiers, consisting of quantities of intermediate inputs, $m_{s-1}(z,q)$, and payments, $t_{s-1}(z,q)$, where s is the tier of the buyer and $s-1$ is the tier of the seller; see equations (A.13), (A.22), (A.30) and (A.45) for quantities and equations (A.15), (A.24), (A.31) and (A.47) for payments. These solutions depend on the productivity of the buyer, z, productivity of the seller, q, the number of suppliers the buyer has, n_s^u, and the aggregate quantity of output the buyer committed to sell to its tier-$s+1$ customers, $M_s(z)$. The payments are proportional to quantities, with the factor of proportionality varying across tiers. And we have shown that in the symmetric case, in which $z = q = 1$ for all firms, these equations acquire a simple recursive structure.

A2 Equilibrium Outcomes

A2.1 Quantities

From (A.30), we have

$$m_{s-1}(z,q) = \tilde{C}_{s-1}(n_s^u)^{\frac{(1-\alpha_s)\gamma_s-1}{\alpha_s}} \frac{M_s(z)}{z} q^{\sigma_s} \quad \text{for } s \in \{2,3,...,S-1\}.$$

This can be expressed as

$$m_{s-1}(z,q) = C_{s-1}\left(n_s^u\right)^{-\frac{1}{\alpha_s}} \frac{M_s(z)}{z} q^{\sigma_s},$$

where

$$C_{s-1} = \left(\frac{B_s}{c_s\gamma_s}\right)^{\frac{\gamma_s}{1-\gamma_s}} H_{s-1}^{-\frac{1}{\alpha_s}}. \tag{A.52}$$

Multiplying both sides of this equation by $n_{s-1}^d f_s(z)$ and integrating over z yields

$$M_{s-1}(q) = n_{s-1}^d C_{s-1}\left(n_s^u\right)^{-\frac{1}{\alpha_s}} q^{\sigma_s} \int_0^\infty \frac{M_s(z)}{z} f_s(z)dz. \tag{A.53}$$

It follows from this equation that we can solve $M_s(q)$ recursively, starting from tier $S-1$, for which (A.45) implies

$$M_{S-1}(q) = n_{S-1}^d C_{S-1} A (n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} H_S q^{\sigma_S}. \tag{A.54}$$

Using (A.53) and (A.54), we obtain

$$\begin{aligned} M_{S-2}(z) &= n_{S-2}^d C_{S-2}(n_{S-1}^u)^{-\frac{1}{\alpha_{S-1}}} z^{\sigma_{S-1}} \int_1^\infty \frac{M_{S-1}(z')}{z'} f_{S-1}(z')dz' \\ &= A n_{S-1}^d n_{S-2}^d C_{S-1} C_{S-2} (n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} (n_{S-1}^u)^{\frac{-1}{\alpha_{S-1}}} z^{\sigma_{S-1}} H_S H_{S-1}. \end{aligned}$$

Continuing the recursion then yields

$$M_s(z) = A n_s^d C_s \left[\prod_{k=s+1}^{S-1} n_k^d \left(n_k^u\right)^{-\frac{1}{\alpha_k}} C_k H_k\right] (n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} z^{\sigma_{s+1}} H_S. \tag{A.55}$$

This formula extends to all $s = 0, 1, ..., S-1$ for $c_0 = 1$ and $\gamma_0 = 1$.

A2.2 Employment

Using the expressions for equilibrium quantities in the previous section, we obtain from (A.32) and (A.55),

$$l_s(z) = \left(\frac{1-\gamma_s}{c_{s-1}\tau_{s-1}\gamma_s}\right)^{\gamma_s-1} (H_{s-1})^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}} (n_s^u)^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}} \frac{M_s(z)}{z} \tag{A.56}$$

$$= \tilde{C}_{s-1}^{\frac{\gamma_s-1}{\gamma_s}} (H_{s-1})^{-\frac{1-\gamma_s}{\alpha_s\gamma_s}} (n_s^u)^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}} \frac{M_s(z)}{z}$$

$$= \frac{c_s\gamma_s}{B_s}\frac{M_s(z)}{z}$$

$$= \frac{c_s\gamma_s}{B_s} A n_s^d C_s H_S \left[\prod_{k=s+1}^{S-1} n_k^d (n_k^u)^{-\frac{1}{\alpha_k}} C_k H_k\right] (n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} z^{\sigma_{s+1}-1},$$

where $B_0 = \gamma_0 = 1$. Multiplying both sides by $\phi_s(r_s)N_s f_s(z)$ and integrating over z provides aggregate employment of firms in tier s of production workers:

$$\begin{aligned} L_{s,m} &= \int_0^\infty \phi_s(r_s)N_s f_s(z)\,dz \\ &= \phi_s(r_s)N_s\frac{c_s\gamma_s}{B_s} A n_s^d C_s \left[\prod_{k=s+1}^{S-1} n_k^d C_k\right]\left[\prod_{k=s+1}^{S-1} (n_k^u)^{-\frac{1}{\alpha_k}} H_k\right] (n_S^u)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} H_S H_s, \\ s &\in \{0,1,...,S-1\}. \end{aligned}$$

For $s = S$, (A.37) and (A.44) imply

$$l_S(z) = A^{\frac{1}{\varepsilon-\gamma_S(\varepsilon-1)}} z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}} \left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-\gamma_S(\varepsilon-1)}} U_S(z)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}} \tag{A.57}$$

$$= A(n_S^u)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} H_{S-1}^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} z^{\varepsilon-1}\left(\frac{1}{c_{S-1}\tau_{S-1}}\right)^{(1-\gamma_S)(\varepsilon-1)}$$

$$\times\left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\varepsilon}\left(\frac{1-\gamma_S}{\gamma_S}\right)^{(1-\gamma_S)(\varepsilon-1)}$$

$$= AC_{S-1}H_{S-1}\frac{\gamma_S}{1-\gamma_S}c_{S-1}\tau_{S-1}(n_S^u)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} z^{\varepsilon-1}.$$

Therefore

$$L_{S,m} = \phi_S(r_S)N_S A C_{S-1}H_{S-1}H_S\frac{\gamma_S}{1-\gamma_S}c_{S-1}\tau_{S-1}(n_S^u)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}}. \tag{A.58}$$

Next note that for $s < S-1$, we have

$$\frac{L_{s-1,m}}{L_{s,m}} = \frac{\phi_{s-1}(r_{s-1})N_{s-1}\gamma_{s-1}B_s c_{s-1}H_{s-1}(n_s^u)^{-\frac{1}{\alpha_s}} n_{s-1}^d C_{s-1}}{\phi_s(r_s)N_s\gamma_s B_{s-1}c_s}.$$

The expressions for c_s and C_{s-1} imply

$$\frac{B_s}{c_s \gamma_s} C_{s-1} = (n_s^u)^{\frac{1-\alpha_s}{\alpha_s}} H_{s-1}^{-1} \frac{1-\gamma_s}{\tau_{s-1} c_{s-1} \gamma_s}, \tag{A.59}$$

and therefore

$$\begin{aligned} \frac{L_{s-1,m}}{L_{s,m}} &= \frac{\phi_{s-1}(r_{s-1}) N_{s-1} n_{s-1}^d}{\phi_s(r_s) N_s n_s^u} \frac{\gamma_{s-1}(1-\gamma_s)}{\gamma_s} \frac{1}{\tau_{s-1} B_{s-1}} \\ &= \frac{\gamma_{s-1}(1-\gamma_s)}{\gamma_s} \frac{1}{\tau_{s-1} B_{s-1}}. \end{aligned} \tag{A.60}$$

Moreover,

$$\begin{aligned} \frac{L_{S-1,m}}{L_{S,m}} &= \frac{\phi_{S-1}(r_{S-1}) N_{S-1} n_{S-1}^d}{\phi_S(r_S) N_S n_S^u} \frac{\gamma_{S-1}(1-\gamma_S)}{\gamma_S B_{S-1} \tau_{S-1}} \\ &= \frac{\gamma_{S-1}(1-\gamma_S)}{\gamma_S} \frac{1}{B_{S-1} \tau_{S-1}}. \end{aligned} \tag{A.61}$$

It follows from this recursion that

$$L_{s,m} = \frac{\gamma_s \Gamma_{s+1}^S}{\gamma_S} \frac{1}{\prod_{j=s}^{S-1} B_j \tau_j} L_{S,m}, \tag{A.62}$$

where $\Gamma_{s+1}^S = \prod_{k=s+1}^S (1-\gamma_k)$. Total employment in manufacturing can therefore be expressed as

$$L_m = C_{L_m} L_{S,m}, \tag{A.63}$$

where

$$C_{L_m} = \left(\frac{1-\gamma_S}{\gamma_S}\right) \left(\frac{\gamma_S}{1-\gamma_S} + \sum_{j=1}^{S-1} \frac{\gamma_j \Gamma_{j+1}^{S-1}}{\prod_{k=j}^{S-1} B_k \tau_k} + \frac{\Gamma_1^{S-1}}{\tau_0 \prod_{k=1}^{S-1} B_k \tau_k}\right). \tag{A.64}$$

Note from (A.62) that aggregate employment in tier s, $s = 0, 1, ..., S-1$, is proportional to aggregate employment in tier S. Moreover, while these factors of proportionality vary across tiers, they do not depend on productivity distributions. This means that the same factors of proportionality hold in the symmetric case discussed in the main text, in which all firms have the same productivity levels $z = q = 1$, as in the case of heterogeneous firms. Finally note that for given investment levels in protective capability and network thickness., aggregate employment in manufacturing, L_m, is fixed, and does not depend on the distributions of productivity levels. Therefore (A.63) and (A.64) imply that aggregate employment in tier S is also independent of productivity distributions. In short, we find that in the symmetric case the aggregate employment level in tier s, $s = 0, 1, ..., S$, is the same as in the case of heterogeneous firms, independently of the tier-specific productivity distributions. Naturally, in the symmetric case all firms in a given tier employ the same number of workers, while in the heterogeneous case employment levels within a tier vary across firms with different productivity levels.

Labor market clearing implies that

$$L-\sum_{s=0}^{S}N_s r_s-\sum_{s=1}^{S}\eta_s N_{s-1}N_s=C_{L_m}L_{S,m}.$$

Using (A.58), this yields a solution to the demand shifter A,

$$A=C_A\frac{L-\sum_{s=0}^{S}N_s r_s-\sum_{s=1}^{S}\eta_s N_{s-1}N_s}{C_{L_m}\phi_S(r_S)N_S}(n_S^u)^{-\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}}(c_{S-1})^{(\varepsilon-1)(1-\gamma_S)}, \qquad (A.65)$$

where

$$C_A=H_{S-1}^{-\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}}\tau_{S-1}^{(1-\gamma_S)(\varepsilon-1)}H_S^{-1}\left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{\gamma_S(\varepsilon-1)-\varepsilon}\left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{-\gamma_S(\varepsilon-1)}\frac{1-\gamma_S}{\gamma_S}.$$

In the symmetric case $H_s=1$ for every tier s. Substituting these values into C_A and the recursive equations for c_s and $\tilde{C}_s$ to obtain an expression for c_{S-1} (see (A.40)), we obtain the value of A for the symmetric case discussed in the text.

A2.3 Welfare

In this section, we derive an expression for equilibrium welfare. Combining equation (A.56) for $l_S(z)$ with (A.45) and the definition of C_{S-1} in (A.46), we obtain:

$$\begin{aligned}
x_S(z) &= AC_{S-1}(n_S^u)^{\frac{\varepsilon(1-\gamma_S)(1-\alpha_S)}{\alpha_S}}z^\varepsilon H_S^{\frac{(1-\gamma_S)+\alpha_S\gamma_S}{\alpha_S}}(c_{S-1}\tau_{S-1})^{\gamma_S}\left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S}\\
&= (H_{S-1})^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1}\left(\frac{1}{c_{S-1}\tau_{S-1}}\right)^{\varepsilon-\gamma_S(\varepsilon-1)}\left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S(\varepsilon-1)}\left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{\varepsilon}\\
&\times A(n_S^u)^{\frac{\varepsilon(1-\gamma_S)(1-\alpha_S)}{\alpha_S}}z^\varepsilon H_S^{\frac{(1-\gamma_S)+\alpha_S\gamma_S}{\alpha_S}}(c_{S-1}\tau_{S-1})^{\gamma_S}\left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S}\\
&= C_x A(n_S^u)^{\frac{\varepsilon(1-\gamma_S)(1-\alpha_S)}{\alpha_S}}(c_{S-1})^{-\varepsilon(1-\gamma_S)}z^\varepsilon,
\end{aligned}$$

where

$$C_x=\left(\frac{1}{\tau_{S-1}}\right)^{\varepsilon(1-\gamma_S)}\left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S\varepsilon}\left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{\varepsilon}H_S^{\frac{(1-\gamma_S)+\alpha_S\gamma_S}{\alpha_S}}(H_{S-1})^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1}$$

is a constant. Therefore

$$x_S(z)^{\frac{\varepsilon-1}{\varepsilon}}=C_x^{\frac{\varepsilon-1}{\varepsilon}}A^{\frac{\varepsilon-1}{\varepsilon}}(n_S^u)^{\frac{(\varepsilon-1)(1-\gamma_S)(1-\alpha_S)}{\alpha_S}}(c_{S-1})^{-(\varepsilon-1)(1-\gamma_S)}z^{\varepsilon-1}$$

and

$$\left(\int_1^\infty x_S(z)^{\frac{\varepsilon-1}{\varepsilon}} f_S(z)dz\right)^{\frac{\varepsilon}{\varepsilon-1}} = C_x A H_S^{\frac{\varepsilon}{\varepsilon-1}} (n_S^u)^{\frac{\varepsilon(1-\gamma_S)(1-\alpha_S)}{\alpha_S}} c_{S-1}^{-\varepsilon(1-\gamma_S)}.$$

Using (A.65) for the demand shifter A, yields

$$\begin{aligned}
&\left(\int_1^\infty x_S(z)^{\frac{\varepsilon-1}{\varepsilon}} f_S(z)dz\right)^{\frac{\varepsilon}{\varepsilon-1}} \\
= \quad & C_X \frac{1}{\phi_S(r_S)N_S}\left[L - \sum_{s=0}^{S} N_s r_s - \sum_{s=1}^{S} \eta_s N_{s-1} N_s\right] (n_S^u)^{\frac{(1-\gamma_S)(1-\alpha_S)}{\alpha_S}} c_{S-1}^{-(1-\gamma_S)},
\end{aligned}$$

where

$$\begin{aligned}
C_X &= \frac{C_x H_S^{\frac{\varepsilon}{\varepsilon-1}} C_A}{C_{L_m}} \qquad \text{(A.66)} \\
&= \tilde{C}_X \frac{\tau_{S-1}^{-(1-\gamma_S)} \frac{\gamma_S}{1-\gamma_S}}{\frac{\gamma_S}{1-\gamma_S} + \sum_{j=1}^{S-1} \frac{\gamma_j \Gamma_{j+1}^{S-1}}{\prod_{k=j}^{S-1} B_k \tau_k} + \frac{\Gamma_1^{S-1}}{\tau_0 \prod_{k=1}^{S-1} B_k \tau_k}},
\end{aligned}$$

and $\tilde{C}_X$ does not depend on $\{\tau_s\}_{s=0}^{S-1}$. This implies a welfare level

$$W = C_X \left[\phi_S(r_S)N_S\right]^{\frac{1}{\varepsilon-1}} \left[L - \sum_{s=0}^{S} N_s r_s - \sum_{s=1}^{S} \eta_s N_{s-1} N_s\right] (n_S^u)^{\frac{(1-\gamma_S)(1-\alpha_S)}{\alpha_S}} c_{S-1}^{-(1-\gamma_S)}.$$

Finally, use the recursive structure of (A.18), (A.29) and (A.40) to obtain[33]

$$c_{S-1}^{-(1-\gamma_S)} = C_K \prod_{s=1}^{S} (n_s^u)^{\frac{\Gamma_s^S(1-\alpha_s)}{\alpha_s}}, \qquad \text{(A.67)}$$

[33] From the definition of c_{S-1} in (A.40) and the recursions inn c_{S-1} and $\tilde{C}_{S-2}$,

$$\begin{aligned}
(c_{S-1})^{-1} &\propto (n_{S-1}^u)^{\frac{(1-\gamma_{S-1})(1-\alpha_{S-1})}{\alpha_{S-1}}} \tilde{C}_{S-2}^{\frac{1-\gamma_{S-1}}{\gamma_{S-1}}} \\
&\propto (n_{S-1}^u)^{\frac{(1-\gamma_{S-1})(1-\alpha_{S-1})}{\alpha_{S-1}}} K_{S-2}^{1-\gamma_{S-1}}.
\end{aligned}$$

Continuing this recursion, we obtain

$$(c_{S-1})^{-1} \propto \prod_{s=1}^{S} (n_s^u)^{\frac{\Gamma_s^{S-1}(1-\alpha_s)}{\alpha_s}},$$

and therefore

$$c_{S-1}^{-(1-\gamma_S)} \propto \prod_{s=1}^{S} (n_s^u)^{\frac{\Gamma_s^S(1-\alpha_s)}{\alpha_s}}.$$

where

$$C_K = \tau_{S-1}^{1-\gamma_S} \left(\frac{1}{\tau_0}\right)^{\Gamma_1^S} \prod_{\ell=1}^{S-1} [H_{\ell-1}]^{\frac{\Gamma_\ell^S(1-\alpha_\ell)}{\alpha_\ell}} \left[\frac{\gamma_\ell}{\tau_\ell B_\ell}\right]^{\Gamma_{\ell+1}^S} \left(\frac{1-\gamma_\ell}{\gamma_\ell}\right)^{\Gamma_\ell^S}. \tag{A.68}$$

Substituting (A.67) into the previous equation then yields

$$W = C_W \left[\phi_S(r_S)N_S\right]^{\frac{1}{\varepsilon-1}} \left[L - \sum_{s=0}^{S} N_s r_s - \sum_{s=1}^{S} \eta_s N_{s-1} N_s\right] \prod_{s=1}^{S} \left[\eta_s \phi_{s-1}(r_{s-1})N_{s-1}\right]^{\frac{\Gamma_s^S(1-\alpha_s)}{\alpha_s}}, \tag{A.69}$$

where

$$C_W := C_X C_K.$$

This is welfare in a market equilibrium for given investment levels in protective capabilities and link formations, and transaction policies $\{\tau_s\}_{s=0}^{S-1}$, where the latter are embodied in the constant C_W.

A2.4 Payoffs to Survivors

Given the solutions to the bargaining problems (without deviants) described in Section **??**, we now characterize the payoffs of surviving firms. From (A.28), a tier $s < S$ supplier with productivity z earns:

$$\pi_s(z) = n_s^d \int_0^\infty t_s(z', z) f_{s+1}(z') dz' - c_s \frac{M_s(z)}{z} \quad \text{for } s \in \{0, 1, ..., S-1\},$$

where $c_0 = 1$. Using (A.15), (A.24), (A.31) and (A.47), we can express this payoff as

$$\begin{aligned} \pi_s(z) &= n_s^d \int_1^\infty \mu_s c_s \frac{m_s(z', z)}{z} f_{s+1}(z') dz' - c_s \frac{M_s(z)}{z} \\ &= (\mu_s - 1) c_s \frac{M_s(z)}{z}. \end{aligned} \tag{A.70}$$

Using (A.55) for $M_s(z)$ then yields

$$\pi_s(z) = (\mu_s - 1) c_s A n_s^d C_s \left[\prod_{j=s+1}^{S-1} n_j^d C_j\right] \left[\prod_{j=s+1}^{S-1} \left(n_j^u\right)^{-\frac{1}{\alpha_j}} H_j\right] \left(n_S^u\right)^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S} - 1} z^{\sigma_{s+1}-1} H_S.$$

To simplify this expression, use (A.59) to obtain

$$\prod_{j=s}^{S-2} C_j = \Gamma_{s+1}^{S-1} \frac{c_{S-1}}{c_s} \prod_{j=s}^{S-2} \left[\left(n_{j+1}^u\right)^{\frac{1-\alpha_{j+1}}{\alpha_{j+1}}} \frac{1}{B_{j+1}\tau_j} H_j^{-1}\right],$$

and substitute this expression into the previous equation. This delivers

$$\pi_s(z) = (\mu_s - 1)\Gamma_{s+1}^{S-1} c_{S-1} C_{S-1} H_S \frac{H_{S-1}}{H_s} \frac{\tau_{S-1} \prod_{j=s}^{S-1} n_j^d}{\tau_s \prod_{j=s+1}^{S-1} n_j^u B_j \tau_j} A(n_S^u)^{\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S}-1} z^{\sigma_{s+1}-1}.$$

Substituting into this equation the expression for C_{S-1} from (A.46) and the expression for A from (A.65), yields

$$\begin{aligned}\pi_s(z) &= (\mu_s - 1) A H_S \frac{\Gamma_{s+1}^{S-1}}{H_s} H_{S-1}^{\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S}} c_{S-1}^{-(\varepsilon-1)(1-\gamma_S)} \left(\frac{1}{\tau_{S-1}}\right)^{\varepsilon-\gamma_S(\varepsilon-1)} \\ &\times \frac{\tau_{S-1} \prod_{j=s}^{S-1} n_j^d}{\tau_s \prod_{j=s+1}^{S-1} n_j^u B_j \tau_j} (n_S^u)^{\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S}-1} z^{\sigma_{s+1}-1} \left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S(\varepsilon-1)} \left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{\varepsilon} \\ &= (\mu_s - 1)\frac{\Gamma_{s+1}^S}{\gamma_S} \frac{z^{\sigma_{s+1}-1}}{n_S^u H_s} \frac{\prod_{j=s}^{S-1} n_j^d}{\tau_s \prod_{j=s+1}^{S-1} n_j^u B_j \tau_j} \frac{L - \sum_{s=0}^S N_s r_s - \sum_{s=1}^S \eta_s N_{s-1} N_s}{C_{L_m} \phi_S(r_S) N_S}.\end{aligned}$$

In equilibrium $n_{s-1}^d = \phi_s(r_s) N_s \eta_s$, and therefore

$$\pi_s(z) = (\mu_s - 1)\frac{\Gamma_{s+1}^S}{\gamma_S} \frac{z^{\sigma_{s+1}-1}}{H_s} \frac{1}{\tau_s \prod_{j=s+1}^{S-1} B_j \tau_j} \frac{L - \sum_{s=0}^S N_s r_s - \sum_{s=1}^S \eta_s N_{s-1} N_s}{C_{L_m} \phi_s(r_s) N_s}.$$

Finally, using the expression for C_{L_m} from (A.64) together with (A.58), we obtain the final formula for this payoff:

$$\begin{aligned}\pi_s(z) &= \frac{L_m}{\phi_s(r_s) N_s} (\mu_s - 1)\Gamma_{s+1}^{S-1} \frac{z^{\sigma_{s+1}-1}}{H_s \tau_s} \frac{1}{\prod_{j=s+1}^{S-1} B_j \tau_j} \\ &\times \frac{1}{\frac{\gamma_S}{1-\gamma_S} + \sum_{j=1}^{S-1} \frac{\gamma_j \Gamma_{j+1}^{S-1}}{\prod_{z=j}^{S-1} B_z \tau_z} + \frac{\Gamma_1^{S-1}}{\tau_0 \prod_{z=1}^{S-1} B_z \tau_z}}.\end{aligned} \tag{A.71}$$

In the case of homogeneous firms, discussed in the main text, $z = H_s = 1$ and the right-hand side of this equation simplifies, because $z^{\sigma_{s+1}-1}/H_s = 1$.

We also need the payoff of a final goods producer in tier S with productivity z. Using (A.38) and the solutions of the bargaining games, we have

$$\pi_S(z) = \pi_S(z) = C_\pi z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}} \left[n_S^u \int_0^\infty m_{S-1}(z,z')^{\alpha_S} f_{S-1}(z') dz'\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}} - T_{S-1}(z),$$

where

$$T_{S-1}(z) = \tau_{S-1} n_S^u \int_0^\infty t_{S-1}(z,z') f_{S-1}(z') dz'$$

Substituting (A.39) and (A.45) into this equation then yields

$$
\begin{aligned}
\pi_S(z) = \pi_S(z) &= C_\pi A^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}} \left(n_S^u\right)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} H_{S-1}^{\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S}} z^{\varepsilon-1} \\
&\times \left(\frac{1}{c_{S-1}\tau_{S-1}}\right)^{(1-\gamma_S)(\varepsilon-1)} \left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{(1-\gamma_S)(\varepsilon-1)} \left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\frac{\gamma_S(\varepsilon-1)^2(1-\gamma_S)}{\varepsilon-\gamma_S(\varepsilon-1)}} - T_{S-1}(z) \\
&= A\left(n_S^u\right)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} H_{S-1}^{\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S}} z^{\varepsilon-1} \\
&\times \left(\frac{1}{c_{S-1}\tau_{S-1}}\right)^{(1-\gamma_S)(\varepsilon-1)} \left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{(1-\gamma_S)(\varepsilon-1)} \left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\gamma_S(\varepsilon-1)} \frac{\varepsilon-\gamma_S(\varepsilon-1)}{\varepsilon} \\
&- T_{S-1}(z) \\
&= AC_{S-1}H_{S-1}c_{S-1}\tau_{S-1}\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(\varepsilon-1)(1-\gamma_S)}\left(n_S^u\right)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} z^{\varepsilon-1} - T_{S-1}(z).
\end{aligned}
$$

Next, using the transfer bill (A.48), we obtain

$$
\pi_S(z) = AC_{S-1}H_{S-1}c_{S-1}\tau_{S-1}\left(n_S^u\right)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} z^{\varepsilon-1}\left[\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(\varepsilon-1)(1-\gamma_S)} - \mu_{S-1}\right], \tag{A.72}
$$

and substituting into this equation the expression for A from (A.65), we obtain

$$
\pi_S(z) = \frac{L_m}{C_{L_m}}\frac{1}{\phi_S(r_S)N_S}\frac{1}{H_S}\frac{1-\gamma_S}{\gamma_S}z^{\varepsilon-1}\left[\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(\varepsilon-1)(1-\gamma_S)} - \mu_{S-1}\right].
$$

Finally, substituting the expression for C_{L_m} from (A.64) yields

$$
\pi_S(z) = \frac{L_m}{\phi_S(r_S)N_S}\frac{z^{\varepsilon-1}}{H_S}\frac{\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(\varepsilon-1)(1-\gamma_S)} - \mu_{S-1}}{\frac{\gamma_S}{1-\gamma_S} + \sum_{j=1}^{S-1}\frac{\gamma_j\Gamma_{j+1}^{S-1}}{\prod_{z=j}^{S-1}B_z\tau_z} + \frac{\Gamma_1^{S-1}}{\tau_0\prod_{z=1}^{S-1}B_z\tau_z}}. \tag{A.73}
$$

This is our final expression for the payoff of a surviving final producer with productivity z. In the case of homogeneous firms $z = H_S = 1$ and the right-hand side of this equation simplifies, because $z^{\varepsilon-1}/H_S = 1$.

A2.5 Ex-Ante Perceived Payoffs

In this subsection, we derive the expected payoffs that firms perceive in the first stage of the game, when they choose investments in protective capability and network thickness. At that point in time all firms in a given tier s are identical, they understand that investment in agility will determine the probability of survival, $\phi_s(r_s)$, and that a productivity z will be drawn from a known distribution with density $f_s(z)$. All firms form rational expectations when making decisions about r_s and η_s.

We derived in Section A1 the solutions to all bilateral bargaining games, where every firm in tier s chooses the same investment in links, η_s, as well as a solution to an off-equilibrium bargaining game in which all firms except for a deviant choose η_s while the deviant firm chooses $\tilde{\eta}_s \neq \eta_s$. Such a deviant has a different number of suppliers in tier $s-1$ compared to the other firms in tier s. For $s \in \{1, 2, ..., S-1\}$, the quantity sold by a deviant firm in tier s with productivity q to a buyer with productivity z in tier $s+1$ is represented by (A.34), which we reproduce as

$$\begin{aligned}\tilde{m}_s(z,q) &= \left(\frac{1-\gamma_{s+1}}{\tau_s\gamma_{s+1}}\right)^{\gamma_{s+1}} (\tilde{c}_s)^{-\frac{1}{1-\alpha_{s+1}}} (c_s)^{-\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{1-\alpha_{s+1}}} \\ &\quad \times (H_s)^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left(n^u_{s+1}\right)^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \frac{M_{s+1}(z)}{z} q^{\sigma_{s+1}},\end{aligned}$$

where c_s is defined in (A.29) and $\tilde{c}_s$ is defined in (A.36). When $\tilde{c}_s = c_s$, i.e., $\tilde{n}^u_s = n^u_s$, this equation yields $\tilde{m}_s(z,q) = m_s(z,q)$, the solution to the bargaining game for a non-deviant firm. Therefore

$$\tilde{m}_s(z,q) = \left(\frac{c_s}{\tilde{c}_s}\right)^{\frac{1}{1-\alpha_{s+1}}} m_s(z,q),$$

and from (A.35),

$$\tilde{t}_s(z,q) = \mu_s \tilde{c}_s \frac{\tilde{m}_s(z,q)}{q}.$$

Using these equations, the payoff to a surviving deviant with productivity z is

$$\begin{aligned}\tilde{\pi}_s &= n^d_s \int_0^\infty \left[\tilde{t}_s(z',z) - \tilde{c}_s \frac{\tilde{m}_s(z',z)}{z}\right] f_{s+1}(z')dz' \\ &= (\mu_s - 1)\, n^d_s \tilde{c}_s \int_0^\infty \frac{\tilde{m}_s(z',z)}{z} f_{s+1}(z')dz' \\ &= (\mu_s - 1)\, n^d_s \int_0^\infty \left(\frac{c_s}{\tilde{c}_s}\right)^{\frac{1}{1-\alpha_{s+1}}} \tilde{c}_s \frac{m_s(z',z)}{z} f_{s+1}(z')dz' \\ &= (\mu_s - 1)\, (c_s)^{\frac{1}{1-\alpha_{s+1}}} \frac{M_s(z)}{z} (\tilde{c}_s)^{-\frac{\alpha_{s+1}}{1-\alpha_{s+1}}}.\end{aligned}$$

From (A.36), $\tilde{c}_s$ is proportional to

$$(\tilde{n}^u_s)^{\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}} = \left[\tilde{\eta}_s \phi_{s-1}(r_{s-1}) N_{s-1}\right]^{\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}}.$$

It follows that

$$\tilde{\pi}_s = \tilde{\pi}_s(z, \tilde{\eta}_s) := \tilde{Q}_{\pi_s}(z)(\tilde{\eta}_s)^{\frac{(1-\alpha_s)(1-\gamma_s)\alpha_{s+1}}{\alpha_s(1-\alpha_{s+1})}} = \tilde{Q}_{\pi_s}(z)(\tilde{\eta}_s)^{\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s-1}},$$

where $\tilde{Q}_{\pi_s}(z)$ depends on the firm's productivity but does not depend on its investment in network thickness. $\tilde{\eta}_s$. Ex-ante, firms do not know z, and therefore their expected payoff is

$$\tilde{\pi}_s(\tilde{\eta}_s) := \mathbb{E}_s\left[\tilde{\pi}_s(z, \tilde{\eta}_s)\right] = Q_{\pi_s}(\tilde{\eta}_s)^{\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s-1}}, \quad s \in \{1, 2, ..., S-1\}, \tag{A.74}$$

where $Q_{\pi_s} := \mathbb{E}_s \left[\tilde{Q}_{\pi_s}(z) \right]$.[34]

Firms in tier 0 make no network choices. A firm in tier 0 anticipates a payoff $\pi_0(z)$ if it draws productivity z, as described in (A.71). Therefore its expected payoff is

$$\pi_0 \quad : \quad = \mathbb{E}_0 \left[\pi_0(z) \right] = \frac{L_m}{\phi_s(r_s) N_s} (\mu_s - 1) \Gamma_{s+1}^{S-1} \frac{1}{\tau_s} \frac{1}{\prod_{j=s+1}^{S-1} B_j \tau_j} \times \frac{1}{\frac{\gamma_S}{1-\gamma_S} + \sum_{j=1}^{S-1} \frac{\gamma_j \Gamma_{j+1}^{S-1}}{\prod_{z=j}^{S-1} B_z \tau_z} + \frac{\Gamma_1^{S-1}}{\tau_0 \prod_{z=1}^{S-1} B_z \tau_z}}. \tag{A.75}$$

Finally, replacing n_S^u with $\tilde{n}_S^u$ in (A.72), we obtain the payoff of a deviant firm in tier S with a productivity draw z; that is

$$\tilde{\pi}_S(z) = A C_{S-1} H_{S-1} c_{S-1} \tau_{S-1} \left(\tilde{n}_S^u \right)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} z^{\varepsilon - 1} \left[\frac{\varepsilon - \gamma_S(\varepsilon-1)}{(\varepsilon-1)(1-\gamma_S)} - \mu_{S-1} \right].$$

Since $\tilde{n}_S^u$ is proportional to $\tilde{\eta}_S$, this firm's expected payoff can be expressed as

$$\tilde{\pi}_S(\tilde{\eta}_S) := \mathbb{E}_S \left[\tilde{\pi}_S(z) \right] = Q_{\pi_S} (\tilde{\eta}_S)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}}, \tag{A.76}$$

where the constant Q_{π_S} does not depend on $\tilde{\eta}_S$.[35]

Our representation of the expected payoffs π_0, $\tilde{\pi}_s(\tilde{\eta}_s)$ and $\tilde{\pi}_S(\tilde{\eta}_S)$ apply to different productivity distributions, including the case of homogeneous firms with $z = 1$ for all. Different distributions of z impact the constants in these equations, such as Q_{π_s} and Q_{π_S}. Therefore they affect the levels of investment in protective capability and network thickness., but, as we show below, they have no impact on optimal policies.

A3 First-Best

The social planner maximizes utility subject to resource constraints. The utility is

[34] A simple way to recover Q_{v_s} is to note that evaluated at $\tilde{\eta}_s = \eta_s$, $Q_{v_s}(\tilde{\eta}_s)^{\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1}}$ equals $\mathbb{E}_s[v_s(z)]$, where $v_s(z)$ is given in (A.71). In other words,

$$Q_{v_s} = \mathbb{E}_s \left[v_s(z) \right] (\eta_s)^{-\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1}}.$$

[35] This constant can be recovered by equating $Q_{v_S}(\tilde{\eta}_S)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}}$, evaluated at $\tilde{\eta}_S = \eta_S$, with $\mathbb{E}_S[v_S(z)]$, where $v_S(z)$ is given in (A.73). In other words,

$$Q_{v_S} = \mathbb{E}_S \left[v_S(z) \right] (\eta_S)^{-\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}}.$$

$$W=\left[\phi_S(r_S)N_S\int_0^\infty x_S(z)^{\frac{\varepsilon-1}{\varepsilon}}f_S(z)dz\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.77}$$

Equation (A.1) together with the solutions to the bargaining games between a final goods producer with productivity z and its suppliers imply the production constraint

$$x_S(z)=zl_S(z)^{\gamma_S}\left[\phi_{S-1}(r_{S-1})N_{S-1}\eta_S\right]^{\frac{1-\gamma_S}{\alpha_S}}\left[\int_0^\infty m_{S-1}\left(z,q\right)^{\alpha_S}f_{S-1}(q)dq\right]^{\frac{1-\gamma_S}{\alpha_S}}. \tag{A.78}$$

In addition, the planner faces a resource constraint

$$L=L_{S,m}+\sum_{s=0}^{S-1}L_{s,m}+\sum_{s=0}^{S}N_sr_s+k\sum_{s=1}^{S}\eta_sN_{s-1}N_s, \tag{A.79}$$

where

$$L_{s,m}=\phi_s(r_s)N_s\int_0^\infty l_s(z)f_s(z)dz\ \text{ for } s\in\{0,1,...,S\},$$

and

$$\begin{aligned}l_s(z)=&\left[\eta_{s+1}\phi_{s+1}(r_{s+1})N_{s+1}\frac{1}{z}\int_0^\infty m_s(z',z)f_{s+1}(z')dz'\right]^{\frac{1}{\gamma_s}}\\&\times\left[\eta_s\phi_{s-1}(r_{s-1})N_{s-1}\int_0^\infty m_{s-1}\left(z,q\right)^{\alpha_s}f_{s-1}(q)dq\right]^{-\frac{1-\gamma_s}{\alpha_s\gamma_s}},\\l_0(z)=&\eta_1\phi_1(r_1)N_1\frac{1}{z}\int_0^\infty m_0(q,z)f_1(q)dq.\end{aligned}$$

The social planner chooses $\{r_s,\eta_s\}$, $\{l_s(z)\}$ and $\{m_s(z,z')\}$ for all z, z' and s, to maximize welfare subject to these constraints.

To simplify the exposition, substitute $l_s(z)$ into the equation for $L_{s,m}$ for $s=0,1,...,S-1$, and the resulting $L_{s,m}$ functions into the labor constraint (A.79), to obtain a single constraint. Next, substitute $x_S\left(z\right)$ into the objective function W. We can then form a Lagrangian for maximizing W subject to a single constraint; that is, the labor constraint. In this problem the choice variables are $\{r_s,\eta_s\}$, $\{l_S\left(z\right)\}$ and $\{m_s(z,z')\}$, and we can use pointwise optimization for the choices of $\{l_S\left(z\right)\}$ and $\{m_s(z,z')\}$ for $s=0,1,...,S-1$.

Letting ω denote the Lagrangian multiplier of the single labor constraint, and denoting by an asterisk the optimal value of a variable, the first-order condition for $l_S\left(z\right)$ yields

$$\frac{\gamma_S\left(W^*\right)^{\frac{1}{\varepsilon}}x_S^*(z)^{\frac{\varepsilon-1}{\varepsilon}}}{l_S^*(z)}=\omega^*\ \text{ for all } z, \tag{A.80}$$

while the first-order condition for $m_{S-1}(z,q)$ yields

$$
\begin{aligned}
&\frac{(W^*)^{\frac{1}{\varepsilon}}\,\phi_S(r_S^*)N_S x_S^*(z)^{\frac{\varepsilon-1}{\varepsilon}}(1-\gamma_S)m_{S-1}^*(z,q)^{\alpha_S-1}}{\int_0^\infty m_{S-1}^*\left(z,z'\right)^{\alpha_S} f_{S-1}(z')dz'} && \text{(A.81)}\\
= \quad &\frac{\omega\phi_{S-1}(r_{S-1}^*)N_{S-1}\frac{1}{\gamma_{S-1}}l_{S-1}^*(q)}{\int_0^\infty m_{S-1}^*(z',q)f_S(z')dz'} \quad \text{for all } z \text{ and } q.
\end{aligned}
$$

Combining (A.80) with (A.81) we obtain

$$
\begin{aligned}
&\frac{\frac{1}{\gamma_S}l_S^*(z)\phi_S(r_S^*)N_S(1-\gamma_S)m_{S-1}^*(z,q)^{\alpha_S-1}}{\int_0^\infty m_{S-1}^*\left(z,z'\right)^{\alpha_S} f_{S-1}(z')dz'} && \text{(A.82)}\\
= \quad &\frac{\phi_{S-1}(r_{S-1}^*)N_{S-1}\frac{1}{\gamma_{S-1}}l_{S-1}^*(q)}{\int_0^\infty m_{S-1}^*(z',q)f_S(z')dz'} \quad \text{for all } z \text{ and } q.
\end{aligned}
$$

Next, the first-order condition with respect to $m_s(z,q)$ for $s \in \{1,2,...,S-2\}$ yields

$$
\begin{aligned}
&\frac{\frac{1}{\gamma_s}\phi_s(r_s^*)N_s l_s^*(q)}{\int_0^\infty m_s^*(z',q)f_{s+1}(z')dz'} && \text{(A.83)}\\
= \quad &\frac{1-\gamma_{s+1}}{\gamma_{s+1}}\phi_{s+1}(r_{s+1}^*)N_{s+1}\frac{l_{s+1}^*(z)m_s^*(z,q)^{\alpha_{s+1}-1}}{\int_1^\infty m_s^*\left(z,z'\right)^{\alpha_{s+1}} f_s(z')dz'} \quad \text{for all } z \text{ and } q,
\end{aligned}
$$

and the first-order condition with respect to $m_0(z,q)$ yields

$$
\phi_0(r_0^*)N_0\eta_1^*\frac{1}{q} = \frac{1-\gamma_1}{\gamma_1}l_1^*(z)\frac{m_0^*(z,q)^{\alpha_1-1}}{\int_0^\infty m_0^*\left(z,z'\right)^{\alpha_1} f_0(z')dz'} \quad \text{for all } z \text{ and } q. \tag{A.84}
$$

From the production function in tier 1 and $x_1(z) = M_1(z)$ (see (A.1)), we obtain

$$
\int_0^\infty m_0^*\left(z,z'\right)^{\alpha_1} f_0(z')dz' = \left[\frac{M_1^*(z)}{z}\right]^{\frac{\alpha_1}{1-\gamma_1}} l_1^*(z)^{-\frac{\gamma_1\alpha_1}{1-\gamma_1}}.
$$

Substituting this equation into (A.84) then yields

$$
m_0^*(z,q) = \left(\frac{1-\gamma_1}{\gamma_1}q\right)^{\frac{1}{1-\alpha_1}} l_1^*(z)^{\frac{1-\gamma_1(1-\alpha_1)}{(1-\gamma_1)(1-\alpha_1)}}\left[\frac{M_1^*(z)}{z}\right]^{-\frac{\alpha_1}{(1-\alpha_1)(1-\gamma_1)}}. \tag{A.85}
$$

Substituting this equation into

$$
l_1^*(z) = \left[\frac{M_1^*(z)}{z}\right]^{\frac{1}{\gamma_1}} U_1^*(z)^{\frac{\gamma_1-1}{\gamma_1}}
$$

where $U_1(z)$ is defined in (A.4) (which results from the production function (A.1)), we obtain:

$$l_1^*(z) = \left(\frac{\gamma_1}{1-\gamma_1}\right)^{1-\gamma_1} (H_0)^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} [\eta_1^* \phi_0(r_0^*) N_0]^{-\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} \frac{M_1^*(z)}{z}. \tag{A.86}$$

Next, substituting this result into (A.85) yields

$$m_0^*(z,q) = \left(\frac{1-\gamma_1}{\gamma_1}\right)^{\gamma_1} (H_0)^{\frac{(1-\alpha_1)\gamma_1-1}{\alpha_1}} [\eta_1^* \phi_0(r_0^*) N_0]^{\frac{(1-\alpha_1)\gamma_1-1}{\alpha_1}} q^{\sigma_1} \frac{M_1^*(z)}{z}. \tag{A.87}$$

This is the optimal quantity required to supply to a firm in tier 1 with productivity z by a firm in tier 0 with productivity q.

We next solve $m_1^*(z,q)$. Substituting $l_1^*(q)/M_1^*(q)$ from (A.86) into the first-order condition (A.83), we obtain

$$m_1^*(z,q) = \left(\frac{1-\gamma_2}{c_1^*\gamma_2}\right)^{\frac{1}{1-\alpha_2}} l_2^*(z)^{\frac{1-\gamma_2(1-\alpha_2)}{(1-\gamma_2)(1-\alpha_2)}} \left[\frac{M_2^*(z)}{z}\right]^{-\frac{\alpha_2}{(1-\alpha_2)(1-\gamma_2)}} q^{\frac{1}{1-\alpha_2}} \tag{A.88}$$

where

$$(c_1^*)^{-1} = \gamma_1 \left(\frac{1-\gamma_1}{\gamma_1}\right)^{1-\gamma_1} (H_0)^{\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}} [\eta_1^* \phi_0(r_0^*) N_0]^{\frac{(1-\alpha_1)(1-\gamma_1)}{\alpha_1}}.$$

Now substitute (A.88) into

$$l_2^*(z) = \left[\frac{M_2^*(z)}{z}\right]^{\frac{1}{\gamma_2}} U_2^*(z)^{\frac{\gamma_2-1}{\gamma_2}},$$

where $U_2(z)$ is defined in (A.4) to obtain

$$l_2^*(z) = \left(\frac{1-\gamma_2}{c_1^*\gamma_2}\right)^{\gamma_2-1} (H_1)^{-\frac{(1-\alpha_2)(1-\gamma_2)}{\alpha_2}} [\eta_2^* \phi_1(r_1^*) N_1]^{-\frac{(1-\alpha_2)(1-\gamma_2)}{\alpha_2}} \frac{M_2^*(z)}{z}. \tag{A.89}$$

Finally, substituting (A.89) into (A.88) yields

$$m_1^*(z,q) = \left(\frac{1-\gamma_2}{c_1^*\gamma_2}\right)^{\gamma_2} (H_1)^{\frac{(1-\alpha_2)\gamma_2-1}{\alpha_2}} [\eta_2^* \phi_1(r_1^*) N_1]^{\frac{(1-\alpha_2)\gamma_2-1}{\alpha_2}} q^{\sigma_2} \frac{M_2^*(z)}{z}. \tag{A.90}$$

Proceeding in similar fashion to solve $m_s^*(z,q)$ for tiers $s \in \{1, 2, ..., S-2\}$, we obtain

$$\begin{aligned} m_s^*(z,q) &= \left(\frac{1-\gamma_{s+1}}{c_s^*\gamma_{s+1}}\right)^{\gamma_{s+1}} (H_s)^{\frac{(1-\alpha_{s+1})\gamma_{s+1}-1}{\alpha_{s+1}}} \\ &\quad \times \left[\eta_{s+1}^* \phi_s(r_s^*) N_s\right]^{\frac{(1-\alpha_{s+1})\gamma_{s+1}-1}{\alpha_{s+1}}} q^{\sigma_{s+1}} \frac{M_{s+1}^*(z)}{z}, \end{aligned} \tag{A.91}$$

where

$$(c_s^*)^{-1} = \gamma_s \left(\frac{1-\gamma_s}{c_{s-1}^*\gamma_s}\right)^{1-\gamma_s} (H_{s-1})^{\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}} \left[\eta_s^* \phi_{s-1}(r_{s-1}^*) N_{s-1}\right]^{\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}}, \tag{A.92}$$

and

$$l_{s+1}^{*}(z) = \left(\frac{1-\gamma_{s+1}}{c_s^{*}\gamma_{s+1}}\right)^{\gamma_{s+1}-1} (H_s)^{-\frac{(1-\alpha_{s+1})(1-\gamma_{s+1})}{\alpha_{s+1}}} \times \left[\eta_{s+1}^{*}\phi_s(r_s^{*})N_s\right]^{-\frac{(1-\alpha_{s+1})(1-\gamma_{s+1})}{\alpha_{s+1}}} \frac{M_{s+1}^{*}(z)}{z}. \tag{A.93}$$

Finally, we solve $m_{S-1}^{*}(z,q)$. Combining (A.82) and

$$l_{S-1}^{*}(z) = \left[\frac{M_{S-1}^{*}(z)}{z}\right]^{\frac{1}{\gamma_2}} U_{S-1}^{*}(z)^{\frac{\gamma_2-1}{\gamma_2}}$$

as we have done above, yields

$$m_{S-1}^{*}(z,q)^{1-\alpha_S} = \frac{1-\gamma_S}{\gamma_S}\gamma_{S-1}\frac{l_S^{*}(z)}{\phi_{S-1}(r_{S-1}^{*})N_{S-1}\eta_S^{*}\int_0^{\infty} m_{S-1}^{*}(z,z')^{\alpha_S} f_{S-1}(z')dz'}\frac{M_{S-1}^{*}(q)}{l_{S-1}^{*}(q)}. \tag{A.94}$$

To complete the solution of the optimal allocation, note that aggregate employment by final good producers is

$$L_{S,m} = \phi_S(r_S)N_S\int_0^{\infty} l_S(z)f_S(z)dz.$$

Substituting $l_S^{*}(z)$ from the first-order condition (A.80) into this equation yields

$$L_{S,m}^{*} = (\omega^{*})^{-1}\gamma_S (W^{*})^{\frac{1}{\varepsilon}}\phi_S(r_S^{*})N_S\int_0^{\infty} x_S^{*}(z)^{\frac{\varepsilon-1}{\varepsilon}} f_S(z)dz.$$

Using this equation and the definition of welfare in (A.77), aggregate employment in tier S can be expressed as

$$L_{S,m}^{*} = (\omega^{*})^{-1}\gamma_S W^{*}. \tag{A.95}$$

Together with (A.80), this yields

$$\begin{aligned} l_S^{*}(z) &= (\omega^{*})^{-1}\gamma_S (W^{*})^{\frac{1}{\varepsilon}} x_S^{*}(z)^{\frac{\varepsilon-1}{\varepsilon}} \\ &= C_{l_S}^{*} x_S^{*}(z)^{\frac{\varepsilon-1}{\varepsilon}}, \end{aligned}$$

where

$$C_{l_S}^{*} := L_{S,m}^{*}(W^{*})^{\frac{1-\varepsilon}{\varepsilon}}. \tag{A.96}$$

Substituting the production function (A.78) into this equation we obtain a solution for the employment level

$$
\begin{aligned}
l_S^*(z) \quad &= \quad \left(C_{l_S}^*\right)^{\frac{\varepsilon}{\varepsilon-\gamma_S(\varepsilon-1)}} z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}} \\
&\times \left[\eta_S^* \phi_{S-1}(r_{S-1}^*) N_{S-1} \int_0^\infty m_{S-1}^*(z,z)^{\alpha_S} f_{S-1}(z') dz'\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}} .
\end{aligned} \tag{A.97}
$$

Next substitute (A.97) together with the solution of $l_{S-1}^*(q)/M_{S-1}^*(q)$ from (A.93) into (A.94), to obtain

$$
\begin{aligned}
m_{S-1}^*(z,q)^{1-\alpha_S} \quad &= \quad \frac{1-\gamma_S}{\gamma_S} q \left(c_{S-1}^*\right)^{-1} \left(C_{l_S}^*\right)^{\frac{\varepsilon}{\varepsilon-\gamma_S(\varepsilon-1)}} z^{\frac{\varepsilon-1}{\varepsilon-\gamma_S(\varepsilon-1)}} \\
&\times \left[\eta_S^* \phi_{S-1}(r_{S-1}^*) N_{S-1} \int_0^\infty m_{S-1}^*(z,z)^{\alpha_S} f_{S-1}(z') dz'\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}-1} ,
\end{aligned} \tag{A.98}
$$

which, using the definition of $U_S(z)$ in (A.4), can be expressed as

$$
\begin{aligned}
m_{S-1}^*(z,q) \quad &= \quad \left(c_{S-1}^*\right)^{-\frac{1}{1-\alpha_S}} q^{\frac{1}{1-\alpha_S}} \left(\frac{1-\gamma_S}{\gamma_S}\right)^{\frac{1}{1-\alpha_S}} \left(C_{l_S}^*\right)^{\frac{\varepsilon}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}} \\
&\times z^{\frac{\varepsilon-1}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}} U_S^*(z)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)-\alpha_S}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}}
\end{aligned} \tag{A.99}
$$

Raising both sides of this equation to the power α_S, multiplying by $n_S^{u,*}$, applying the operator $\mathbb{E}_{S-1}$ to both sides and raising the outcomes of both sides to the power $1/\alpha_S$, yields

$$
\begin{aligned}
U_S^*(z) \quad &= \quad H_{S-1}^{\frac{1}{\alpha_S}} \left(\frac{1-\gamma_S}{c_{S-1}^*\gamma_S}\right)^{\frac{1}{1-\alpha_S}} \left(C_{l_S}^*\right)^{\frac{\varepsilon}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}} \left(n_S^{u,*}\right)^{\frac{1}{\alpha_S}} \\
&\times z^{\frac{\varepsilon-1}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}} U_S^*(z)^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)-\alpha_S}{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}} .
\end{aligned}
$$

This equation provides the solution

$$
\begin{aligned}
U_S^*(z) \quad &= \quad H_{S-1}^{\frac{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}{\alpha_S}} \left(\frac{1-\gamma_S}{c_{S-1}^*\gamma_S}\right)^{\varepsilon-\gamma_S(\varepsilon-1)} \left(C_{l_S}^*\right)^{\varepsilon} \\
&\times z^{\varepsilon-1} (n_S^{u,*})^{\frac{(1-\alpha_S)[\varepsilon-\gamma_S(\varepsilon-1)]}{\alpha_S}} .
\end{aligned} \tag{A.100}
$$

Finally, substituting (A.100) into (A.99), we obtain the solution of the optimal transaction quantity between a buyer in tier S with productivity z and a supplier in tier $S-1$ with productivity q:

$$
m_{S-1}^*(z,q) = C_{S-1}^* (n_S^{u,*})^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} q^{\sigma_S} z^{\varepsilon-1}, \tag{A.101}
$$

where

$$C_{S-1}^{*}=\left(\frac{1-\gamma_{S}}{\gamma_{S}}\right)^{\varepsilon-\gamma_{S}(\varepsilon-1)}\left[H_{S-1}\right]^{\frac{(1-\alpha_{S})(1-\gamma_{S})(\varepsilon-1)}{\alpha_{S}}-1}\left(C_{l_{S}}^{*}\right)^{\varepsilon}\left(c_{S-1}^{*}\right)^{-[\varepsilon-\gamma_{S}(\varepsilon-1)]}$$

Finally, using (A.97), we obtain the employment levels of firms with varying productivity levels

$$\begin{aligned}l_{S}^{*}(z) &= \left(C_{l_{S}}^{*}\right)^{\frac{\varepsilon}{\varepsilon-\gamma_{S}(\varepsilon-1)}}\left(C_{S-1}^{*}\right)^{\frac{(1-\gamma_{S})(\varepsilon-1)}{\varepsilon-\gamma_{S}(\varepsilon-1)}}\left(H_{S-1}\right)^{\frac{(1-\gamma_{S})(\varepsilon-1)}{\alpha_{S}[\varepsilon-\gamma_{S}(\varepsilon-1)]}}\left[n_{S}^{u,*}\right]^{\frac{(1-\gamma_{S})(\varepsilon-1)(1-\alpha_{S})}{\alpha_{S}}}(z)^{\varepsilon-1} \qquad (A.102)\\ &= \left(C_{l_{S}}^{*}\right)^{\varepsilon}\left(\frac{1-\gamma_{S}}{c_{S-1}^{*}\gamma_{S}}\right)^{(1-\gamma_{S})(\varepsilon-1)} H_{S-1}^{\frac{(1-\alpha_{S})(\varepsilon-1)(1-\gamma_{S})}{\alpha_{S}}}\left[n_{S}^{u,*}\right]^{\frac{(1-\gamma_{S})(\varepsilon-1)(1-\alpha_{S})}{\alpha_{S}}}(z)^{\varepsilon-1}\\ &= C_{S-1}^{*}\frac{c_{S-1}^{*}\gamma_{S}}{1-\gamma_{S}}H_{S-1}\left[n_{S}^{u,*}\right]^{\frac{(1-\gamma_{S})(\varepsilon-1)(1-\alpha_{S})}{\alpha_{S}}}(z)^{\varepsilon-1}.\end{aligned}$$

We have so far derived optimal transaction levels between firms in adjacent tiers as functions of their productivity levels, and optimal employment levels of firms in different tiers as functions of productivity levels. The solution to $m_{s-1}^{*}(z,q)$, i.e., the supply of inputs by a firm in tier $s-1$ with productivity q to a firm in tier s with productivity z, also depends on the optimal aggregate production and sales of the firm in tier s, $M_{s}^{*}(z)$. Therefore, in order to complete the solution we need solutions to $\{M_{s}^{*}(z)\}$, which we provide next. Note, however, that our formulas hold for arbitrary distributions of productivity, including the case of homogeneous firms where $z=q=1=H_{s}$ for all firms and all tiers. In this case $M_{s}^{*}=n_{s}^{d,*}m_{s-1}^{*}$ is the same for every firm in tier s and m_{s-1}^{*} represents the supply of inputs by a tier-$s-1$ firm to a tier-s firm. In this case our formulas represent a recursive system in m_{s-1}^{*} that provides a complete solution to the optimal transaction levels. For the heterogeneous case we can derive a recursive system in the values of $M_{s}^{*}(z)$ in order to obtain solutions to $\{M_{s}^{*}(z)\}$. Once these solutions are available, we can substitute them into the equations for $m_{s-1}^{*}(z,q)$ in order to obtain final solutions to the firm-to-firm transaction levels.

Using (A.87), (A.91) and (A.92), we obtain

$$m_{s}^{*}(z,q)=C_{s}^{*}\left(n_{s+1}^{u,*}\right)^{-\frac{1}{\alpha_{s+1}}}q^{\sigma_{s+1}}\frac{M_{s+1}^{*}(z)}{z}\quad\text{for } s\in\{0,1,...,S-2\}, \qquad (A.103)$$

where

$$C_{s}^{*}:=\left(\frac{1}{c_{s+1}^{*}\gamma_{s+1}}\right)^{\frac{\gamma_{s+1}}{1-\gamma_{s+1}}}H_{s}^{-\frac{1}{\alpha_{s+1}}}.$$

Next, substituting (A.101) into

$$M_{S-1}^{*}(z)=n_{S-1}^{d,*}\int_{0}^{\infty}m_{S-1}^{*}(z,q)f_{S-1}(q)\,dq,$$

we obtain

$$M_{S-1}^{*}(z)=C_{S-1}^{*}n_{S-1}^{d,*}z^{\sigma_{S}}(n_{S}^{u,*})^{\frac{(1-\alpha_{S})(1-\gamma_{S})(\varepsilon-1)}{\alpha_{S}}-1}H_{S}. \qquad (A.104)$$

To iterating further, substitute (A.103) for $s = S - 2$ into

$$M_{S-2}^{*}(z) = n_{S-2}^{d,*} \int_{0}^{\infty} m_{S-2}^{*}(z,q) f_{S-2}(q)\, dq,$$

and substitute (A.104) into the result to obtain

$$M_{S-2}^{*}(z) = C_{S-1}^{*} C_{S-2}^{*} n_{S-1}^{d,*} n_{S-2}^{d,*} (n_{S}^{u,*})^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} \left(n_{S-1}^{u,*}\right)^{-\frac{1}{\alpha_{S-1}}} H_S H_{S-1} z^{\sigma_{S-1}}.$$

Continuing this recursion, we get

$$M_{s}^{*}(z) = z^{\sigma_{s+1}} (n_{S}^{u,*})^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} \prod_{j=s+1}^{S-1} \left(n_{j}^{u,*}\right)^{-\frac{1}{\alpha_j}} \times \prod_{j=s}^{S-1} C_{j}^{*} n_{j}^{d,*} H_{j+1} \quad \text{for } s \in \{0,1,...,S-2\}. \tag{A.105}$$

Together with (A.104), this equation provides a solution to the aggregate production and sales of every type of firm in every tier. To obtain these variable in the symmetric case, we substitute into these equations $z = H_s = 1$ for all firms and all tiers.

Next, use the labor formulas, such as (A.93), together with the above formulas for $M_s^*(z)$, to obtain the solutions

$$l_{s}^{*}(z) = c_{s}^{*} \gamma_{s} z^{\sigma_{s+1}-1} (n_{S}^{u,*})^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} \prod_{j=s+1}^{S-1} \left(n_{j}^{u,*}\right)^{-\frac{1}{\alpha_j}} \times \prod_{j=s}^{S-1} C_{j}^{*} n_{j}^{d,*} H_{j+1} \quad \text{for } s \in \{0,1,...,S-1\} \tag{A.106}$$

and

$$L_{s,m}^{*} = \phi_s(r_s^{*}) N_s c_{s}^{*} \gamma_{s} (n_{S}^{u,*})^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} \prod_{j=s+1}^{S-1} \left(n_{j}^{u,*}\right)^{-\frac{1}{\alpha_j}} \times H_S \prod_{j=s}^{S-1} C_{j}^{*} n_{j}^{d,*} H_{j} \quad \text{for } s \in \{0,1,...,S-1\}. \tag{A.107}$$

Therefore

$$
\begin{aligned}
\frac{L^*_{s-1,m}}{L^*_{s,m}} &= \frac{\phi_{s-1}(r^*_{s-1})N_{s-1}}{\phi_s(r^*_s)N_s} c^*_{s-1}\gamma_{s-1}C^*_{s-1}H_{s-1}n^{d,*}_{s-1}\frac{1}{c^*_s\gamma_s}\left(n^{u,*}_s\right)^{-\frac{1}{\alpha_s}} \\
&= \frac{\phi_{s-1}(r^*_{s-1})N_{s-1}}{\phi_s(r^*_s)N_s}\frac{n^{d,*}_{s-1}}{n^{u,*}_s}\frac{1-\gamma_s}{\gamma_s}\gamma_{s-1} \\
&= \frac{1-\gamma_s}{\gamma_s}\gamma_{s-1}
\end{aligned} \tag{A.108}
$$

Moreover, from (A.102) we obtain:

$$
L^*_{S,m} = \phi_S(r^*_S)N_S C^*_{S-1}\frac{c^*_{S-1}\gamma_S}{1-\gamma_S}\left[n^{u,*}_S\right]^{\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}} H_S H_{S-1}
$$

and therefore

$$
\begin{aligned}
\frac{L^*_{S-1,m}}{L^*_{S,m}} &= \frac{\phi_{S-1}(r^*_{S-1})N_{S-1}}{\phi_S(r^*_S)N_S}\gamma_{S-1}\frac{n^{d,*}_{S-1}}{n^{u,*}_S}\frac{1-\gamma_S}{\gamma_S} \\
&= \frac{1-\gamma_S}{\gamma_S}\gamma_{S-1}.
\end{aligned} \tag{A.109}
$$

Note that optimal relative employment levels across tiers depend only on the Cobb-Douglas coefficients of the production functions, which are exogenous. Together with the labor constraint (A.79), these ratios imply

$$
L^*_{S,m} = \gamma_S\left[L - \sum_{s=0}^{S} N_s r^*_s - k\sum_{s=1}^{S}\eta^*_s N_{s-1}N_s\right]. \tag{A.110}
$$

In other words, the final good producers employ a fraction γ_S of manufacturing labor and producers of intermediate goods in every tier $s < S$ also employ fixed fractions. For example, (A.109) and (A.110) imply that aggregate labor use in tier $S-1$ is a fraction $(1-\gamma_S)\,\gamma_{S-1}$ of manufacturing employment. Using (A.96), (A.110) and the definition of W in (A.77), we obtain

$$
C^*_{l_S} = \frac{\gamma_S\left[L - \sum_{s=0}^{S} N_s r^*_s - k\sum_{s=1}^{S}\eta^*_s N_{s-1}N_s\right]}{\phi_S(r^*_S)N^*_S\int_1^\infty x^*_S(z)^{\frac{\varepsilon-1}{\varepsilon}} f_S(z)dz}.
$$

Together with the production function (A.1) and the solutions to $m^*_{S-1}(z,q)$ this provides a solution to $C^*_{l_S}$ as a function of the investment levels $\{r^*_s\}$ and $\{\eta^*_s\}$.

Next, consider the social planner's choice of protective capability and network formation. The first-order condition with respect to r^*_S is

$$
\frac{\varepsilon}{\varepsilon-1}W^*\frac{\phi'_S(r^*_S)r^*_S}{\phi_S(r^*_S)} = \omega^*\left[\frac{\phi'_S(r^*_S)r^*_S}{\phi'_S(r^*_S)}L^*_{S,m} + \frac{1}{\gamma_{S-1}}\frac{\phi'_S(r^*_S)r^*_S}{\phi'_S(r^*_S)}L^*_{S-1,m} + N_S r^*_S\right].
$$

Combining with (A.95) and (A.109), this yields

$$\frac{1}{\varepsilon-1}\frac{\phi_S'(r_S^*)r_S^*}{\phi_S(r_S^*)}=\frac{N_S r_S^*}{L-\sum_{s=0}^{S}N_s r_s^*-k\sum_{s=1}^{S}\eta_s^* N_{s-1}N_s}. \tag{A.111}$$

The first-order condition with respect to r_s^* for $s<S$ is

$$\frac{\phi_s'(r_s^*)r_s^*}{\phi_s(r_s^*)}\left[L_{s,m}^*+\frac{1}{\gamma_{s-1}}L_{s-1,m}^*-\frac{1-\gamma_{s+1}}{\alpha_{s+1}\gamma_{s+1}}L_{s+1,m}^*\right]=N_s r_s^*,$$

which together with (A.108), (A.109) and (A.110) implies

$$\frac{\Gamma_{s+1}^S(1-\alpha_{s+1})}{\alpha_{s+1}}\frac{\phi_s'(r_s^*)r_s^*}{\phi_s(r_s^*)}=\frac{N_s r_s^*}{L-\sum_{s=0}^{S}N_s r_s^*-k\sum_{s=1}^{S}\eta_s^* N_{s-1}N_s}. \tag{A.112}$$

Using a similar approach, the first-order conditions with respect to η_s^* for $s\in\{1,2,...,S\}$ imply

$$\frac{(1-\gamma_S)(1-\alpha_S)}{\alpha_S}=\frac{kN_{S-1}N_S\eta_S^*}{L-\sum_{s=0}N_s r_s^*-k\sum_{s=1}\eta_s^* N_{s-1}N_s} \tag{A.113}$$

$$\frac{\Gamma_{s+1}^S(1-\alpha_{s+1})}{\alpha_{s+1}}=\frac{kN_sN_{s+1}\eta_{s+1}^*}{L-\sum_{s=0}^{S}N_s r_s^*-k\sum_{s=1}\eta_s^* N_{s-1}N_s}\quad\text{for } s=\{0,1,...,S-2\}. \tag{A.114}$$

Note that(A.111)-(A.114) provide solutions to the optimal investment levels and these solutions do not depend on the degree of productivity heterogeneity in the various tiers. For this reason they also represent the solutions for the homogeneous case.

A4 First-Best Policies

We have characterized the optimal allocation in the previous section. We now show that there is a set of policies that implement this allocation.

First note that optimal subsidies to transactions can be derived from a comparison of the equilibrium allocation of labor to tiers, given by (A.60) and (A.61), with the optimal allocation of labor to tiers, given by (A.108) and (A.109). This comparison yields the first-best transaction subsidies

$$\tau_s^*=\frac{1}{B_s}\qquad\text{for } s\in\{1,2,...,S-1\}, \tag{A.115}$$

$$\tau_0^*=1, \tag{A.116}$$

where, recall,

$$B_s = \mu_{s-1}(1-\gamma_s)+\gamma_s>1,$$
$$\mu_{s-1} = \beta_s+(1-\beta_s)\frac{\sigma_s}{\sigma_s-1}>1.$$

While these subsidies secure the optimal allocation of labor to tiers, the remaining question is whether they also secure the optimal allocation of labor across firms with different productivity levels within tiers. In the homogeneous case this is not an issues, because within a tier all firms have the same productivity.

For firms with varying productivity levels, (A.2), (A.86) and (A.93) imply that the ratio of optimal employment levels by firms with productivities z and z' is

$$\frac{l_s^*(z)}{l_s^*(z')} = \frac{M_s^*(z)\,z'}{M_s^*(z')\,z}, \quad \text{for } s \in \{0,1,2,...,S-1\}.$$

However, (A.104) and (A.105) imply that

$$\frac{M_s^*(z)\,z'}{M_s^*(z')\,z} = \left(\frac{z}{z'}\right)^{\sigma_{s+1}-1}, \quad \text{for } s \in \{0,1,2,...,S-1\}.$$

It follows that the optimal allocation of labor across firms in a given tier s depends on the ratio of their productivity levels and the elasticity of substitution across inputs of the firms in the tier above them.

Next note from (A.2) and the equilibrium allocation of labor across firms in a given tier s, (A.32), that

$$\frac{l_s(z)}{l_s(z')} = \frac{M_s(z)\,z'}{M_s(z')\,z}, \quad \text{for } s \in \{0,1,2,...,S-1\},$$

while from (A.55),

$$\frac{M_s(z)\,z'}{M_s(z')\,z} = \left(\frac{z}{z'}\right)^{\sigma_{s+1}-1}, \quad \text{for } s \in \{0,1,2,...,S-1\}.$$

Therefore in equilibrium the relative labor use of firms in a given tier s is the same as in the optimal allocation. Since the transaction subsidies (A.115) and (A.116) ensure optimal aggregate employment in every tier, these policies also ensure the optimal distribution of these employment levees across firm within the tiers.

We next turn to subsidies for investment in protective capability and link formation. A tier-s firm chooses

$$(r_s, \eta_s) = \arg\max_{\tilde{r}_s, \tilde{\eta}_s} \phi_s(\tilde{r}_s)\,\tilde{\pi}_s(\tilde{\eta}_s) - \theta_s \tilde{r}_s - \vartheta_s k \tilde{\eta}_s N_{s-1} \quad \text{for } s \in \{1,2,...,S\} \tag{A.117}$$

and

$$r_0 = \arg\max_{\tilde{r}_0} \phi_0(\tilde{r}_0)\,\pi_0 - \theta_0 \tilde{r}_0 \quad \text{for } s = 0,$$

where the expressions for π_0 and $\tilde{\pi}_s(\tilde{\eta}_s)$, $s = 1,2,...,S$, are given by (A.74)-(A.76). The first-order conditions for the choice of protective capabilities are therefore

$$\phi_s'(r_s)\tilde{\pi}_s(\eta_s) = \theta_s \quad \text{for } s \in \{0,1,2,...,S\}, \tag{A.118}$$

where $\tilde{\pi}_s(\tilde{\eta}_s)$ is evaluated at the equilibrium level of links, $\tilde{\eta}_s = \eta_s$, and $\tilde{\pi}_0(\eta_0) := \pi_0$. From the equilibrium values of $\pi_s(z)$ in (A.71) and (A.73), we obtain

$$\begin{aligned}\tilde{\pi}_s(\eta_s) \quad &= \quad \mathbb{E}_s\left[\pi_s(z)\right] = \frac{L_m}{\phi_s(r_s)N_s}(\mu_s - 1)\Gamma_{s+1}^{S-1}\frac{1}{\tau_s}\frac{1}{\prod_{j=s+1}^{S-1}B_j\tau_j} \\ &\times \frac{1}{\frac{\gamma_S}{1-\gamma_S} + \sum_{j=1}^{S-1}\frac{\gamma_j\Gamma_{j+1}^{S-1}}{\prod_{z=j}^{S-1}B_z\tau_z} + \frac{\Gamma_1^{S-1}}{\tau_0\prod_{z=1}^{S-1}B_z\tau_z}} \quad \text{for } s \in \{1,2,...,S-1\}\end{aligned} \tag{A.119}$$

$$\tilde{\pi}_S(\eta_S) = \mathbb{E}_S\left[\pi_S(z)\right] = \frac{L_m}{\phi_S(r_S)N_S}\frac{\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(\varepsilon-1)(1-\gamma_S)} - \mu_{S-1}}{\frac{\gamma_S}{1-\gamma_S} + \sum_{j=1}^{S-1}\frac{\gamma_j\Gamma_{j+1}^{S-1}}{\prod_{z=j}^{S-1}B_z\tau_z} + \frac{\Gamma_1^{S-1}}{\tau_0\prod_{z=1}^{S-1}B_z\tau_z}}, \tag{A.120}$$

where, recall,

$$L_m = L - \sum_{s=0}^{S} N_s r_s - k\sum_{s=1}^{S}\eta_s N_{s-1}N_s$$

is manufacturing employment. To implement the optimal allocation, the planner uses optimal transaction subsidies τ_s^*, which satisfy $B_s\tau_s^* = 1$ for all tiers, with $B_0 = 1$. In this case $\tilde{\pi}_s(\eta_s)$ and $\tilde{\pi}_S(\eta_S)$ become

$$\tilde{\pi}_s(\eta_s) = \mathbb{E}_s\left[\pi_s(z)\right] = \frac{L_m}{\phi_s(r_s)N_s}(\mu_s - 1)\Gamma_{s+1}^{S}B_s \quad \text{for } s \in \{1,2,...,S-1\}, \tag{A.121}$$

$$\tilde{\pi}_S(\eta_S) = \mathbb{E}_S\left[\pi_S(z)\right] = \frac{L_m}{\phi_S(r_S)N_S}\left[\frac{\varepsilon - \gamma_S(\varepsilon-1)}{(\varepsilon-1)(1-\gamma_S)} - \mu_{S-1}\right](1-\gamma_S). \tag{A.122}$$

Substituting these equations into the first-order conditions (A.118), and recalling that $\mu_s = \beta_{s+1} + \left(1-\beta_{s+1}\right)\sigma_{s+1}/\left(\sigma_{s+1}-1\right)$ and $\sigma_s = 1/\left(1-\alpha_s\right)$, we obtain

$$\begin{aligned}\frac{\Gamma_{s+1}^{S}\phi_s'(r_s)r_s}{\phi_s(r_s)}\frac{1-\alpha_{s+1}}{\alpha_{s+1}}\left(1-\beta_{s+1}\right)B_s \quad &= \quad \frac{\theta_s r_s N_s}{L - \sum_{s=0}^{S}N_s r_s - k\sum_{s=1}^{S}\eta_s N_{s-1}N_s} \\ \text{for } s \quad &\in \quad \{0,1,2,...,S-1\},\end{aligned}$$

$$\frac{\phi_S'(r_S)r_S}{\phi_S(r_S)}\left[\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(\varepsilon-1)(1-\gamma_S)} - \mu_{S-1}\right](1-\gamma_S) = \frac{\theta_S r_S N_S}{L - \sum_{s=0}^{S}N_s r_s - k\sum_{s=1}^{S}\eta_s N_{s-1}N_s}.$$

Comparing the first of these equations, for $s \in \{0,1,2,...,S-1\}$, to the optimum condition (A.112), we find that the optimal policy is

$$\theta_s^* = \frac{1-\beta_{s+1}}{\tau_s^*} \quad \text{for } s \in \{0,1,...,S-1\}. \tag{A.123}$$

And comparing the second of these conditions, for $s = S$, to the optimum condition (A.111), we find that

$$\theta_S^* = 1 - \frac{(1-\beta_S)(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1}. \tag{A.124}$$

These optimal policies apply to all productivity distributions. They therefore also apply to the case of homogeneous productivities. Since $\sigma_S > \varepsilon$, (A.124) implies $\theta_S^* \in (0,1)$. That is, it is optimal to subsidize investment in the protective capabilities of final goods producers. As for manufacturers of intermediate inputs, (A.123) implies $\theta_0^* \in (0,1)$, because $\tau_0^* = 1$, so that it is also optimal to subsidize investment in the protective capabilities of the most upper tier firms. But for firms in intermediate tiers, i.e., $s = 1, 2, ..., S-1$, it is optimal to subsidize investment in the protective capabilities of firms in tier s if and only if $\left(1-\beta_{s+1}\right) B_s < 1$. In other words, there may exist tiers in which the optimal policy is to tax investment in their protective capabilities, because $\left(1-\beta_{s+1}\right) B_s > 1$.

Now consider subsidies to investment in link formation, $\{\vartheta_s\}_{s=1}^S$. For firms in tier s, $s \in \{1, 2, ..., S-1\}$, (A.117) provides a solution for the choice of $\tilde{\eta}_s$. Using (A.74), the first-order condition yields

$$\phi_s(r_s)\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s-1}\frac{\tilde{\pi}_s(\eta_s)}{\eta_s} = \vartheta_s k N_{s-1},$$

where r_s and η_s are the equilibrium choices. Together with the first-order condition for r_s, given by (A.118), this yields

$$\frac{r_s}{\eta_s} = \frac{\vartheta_s}{\theta_s} k N_{s-1} \frac{\sigma_s - 1}{(1-\gamma_s)(\sigma_{s+1}-1)} \frac{\phi_s'(r_s) r_s}{\phi_s(r_s)}. \tag{A.125}$$

Next, substituting into this equation the condition for an optimal choice of r_s, given by (A.112), to obtain a condition that has to be satisfied by the optimal policies θ_s^* and ϑ_s^*:

$$\Gamma_{s+1}^S \frac{(1-\gamma_s)(1-\alpha_s)}{\alpha_s}\theta_s^* = \frac{\vartheta_s^* k \eta_s^* N_{s-1} N_s}{L - \sum_{s=0}^S N_s r_s^* - k \sum_{s=1}^S \eta_s^* N_{s-1} N_s}.$$

However, from (A.114) we have

$$\frac{\Gamma_s^S (1-\alpha_s)}{\alpha_s} = \frac{k \eta_s^* N_{s-1} N_s}{L - \sum_{s=0}^S N_s r_s^* - k \sum_{s=1} \eta_s^* N_{s-1} N_s},$$

and the last two equations imply

$$\vartheta_s^* = \theta_s^* = \frac{1-\beta_{s+1}}{\tau_s^*} \quad \text{for } s \in \{1, 2, ..., S-1\}.$$

Using (A.76), for a firm in tier S the solution to the maximization problem (A.117) yields the first-order condition

$$\phi_S(r_S)\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}\frac{\tilde{\pi}_s(\eta_s)}{\eta_S} = \vartheta_S k N_{S-1}.$$

Combining with (A.118), this can be expressed as

$$\frac{\phi_S(r_S)}{\phi_S'(r_S) r_S}\frac{(1-\gamma_S)(\varepsilon-1)(1-\alpha_S)}{\alpha_S}\theta_S r_S N_S = \vartheta_S k \eta_S N_{S-1} N_S.$$

Now substitute into this equation the condition for an optimal choice of r_S, given by (A.111), to obtain

$$\frac{(1-\gamma_S)(1-\alpha_S)}{\alpha_S}\theta_S^* = \frac{\vartheta_S^*\eta_S^* N_{S-1}N_S}{L-\sum_{s=0}^S N_s r_s^* - k\sum_{s=1}^S \eta_s^* N_{s-1}N_s}.$$

Finally, substituting into this equation the condition for an optimal choice of η_S, (A.113), we obtain

$$\vartheta_S^* = \theta_S^* = 1 - \frac{(1-\beta_S)(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1}.$$

In summary, in every tier the first-best subsidy to investment in link formation is the same as the first-best subsidy to investment in protective capabilities.

A5 Second-Best Policies

In this section, we characterize the second-best policies for protective capabilities and link formations, $\{\theta_s^\circ\}_{s=0}^S$ and $\{\vartheta_s^\circ\}_{s=1}^S$ respectively. In the second-best world the policy maker cannot use transaction subsidies, so that $\tau_s^\circ = 1$ in every tier s.

Equilibrium welfare is represented by (A.69), which we reproduce here for convenience:

$$W = C_W\left[\phi_S(r_S)N_S\right]^{\frac{1}{\varepsilon-1}}\left(L - \sum_{s=0}^S N_s r_s - \sum_{s=1}^S \eta_s N_{s-1}N_s\right)\prod_{s=1}^S\left[\eta_s\phi_{s-1}(r_{s-1})N_{s-1}\right]^{\frac{\Gamma_s^S(1-\alpha_s)}{\alpha_s}}.$$

Recall that this equation holds for arbitrary values of transaction subsidies, $\{\tau_s\}_{s=0}^{S-1}$, which are embodied in the constant C_W. In the first-best the transaction subsidies are $\{\tau_s^*\}_{s=0}^{S-1}$, while in the second-best they all equal one. In both cases the planner chooses $\{r_s\}_{s=0}^S$ and $\{\eta_s\}_{s=1}^S$ to maximize this welfare function. It follows that the second-best investment levels are the same as the first-best investment levels. That is,

$$r_s^\circ = r_s^* \text{ for } s \in \{0,1,...,S\},$$

and

$$\eta_s^\circ = \eta_s^* \text{ for } s \in \{1,2,...,S\}.$$

This implies that conditions (A.111)-(A.114) are satisfied in the second-best allocation.[36]

While the optimal investment levels are the same in the first- and second-best, the policies that implement them differ. The difference arises from the fact that the ex-ante expected payoffs of the firms differ in these regimes. Whereas (A.119) and (A.120) imply (A.121) and (A.122) in the first-best, in the second-best they imply

[36] Note also that we can obtain the optimal transaction subsidies by choosing $\{\tau_s\}_{s=0}^{S-1}$ that maximize $C_W = C_X C_K$, where C_X is defined in (A.66) and C_K is defined in (A.68).

$$\tilde{\pi}_s(\eta_s) = \frac{L_m}{\phi_s(r_s)N_s}(\mu_s - 1)\Gamma_{s+1}^S \frac{1}{J\prod_{j=s+1}^{S-1} B_j} \quad \text{for } s \in \{1, 2, ..., S-1\}\,, \tag{A.126}$$

$$\tilde{\pi}_S(\eta_S) = \mathbb{E}\left[\pi_S(z)\right] = \frac{L_m}{J\phi_S(r_S)N_S}\left[\frac{\varepsilon - \gamma_S(\varepsilon - 1)}{(\varepsilon - 1)(1 - \gamma_S)} - \mu_{S-1}\right](1 - \gamma_S)\,, \tag{A.127}$$

where[37]

$$J := \gamma_S + \sum_{j=1}^{S-1} \frac{\gamma_j \Gamma_{j+1}^S}{\prod_{s=j}^{S-1} B_s} + \frac{\Gamma_1^S}{\prod_{s=1}^{S-1} B_s} < 1.$$

Following the steps we used in the analysis of the first-best policies, we now obtain

$$\theta_0^\circ = \frac{1 - \beta_1}{J\prod_{s=1}^{S-1} B_s}, \tag{A.128}$$

$$\theta_s^\circ = \vartheta_s^\circ = \frac{1 - \beta_{s+1}}{J\prod_{j=s+1}^{S-1} B_j} \text{ for } s \in \{1, 2, ..., S-1\}, \tag{A.129}$$

$$\theta_S^\circ = \vartheta_S^\circ = \frac{1}{J}\left[1 - \frac{(1 - \beta_S)(1 - \gamma_S)(\varepsilon - 1)}{\sigma_S - 1}\right]. \tag{A.130}$$

These policy measures can be subsidies or taxes for tiers $s \in \{1, 2, ..., S\}$, but they imply that protective capabilities have to be subsidized in tier 0. This can be seen from

$$J\prod_{s=1}^{S-1} B_s = \gamma_S \prod_{s=1}^{S-1} B_s + \sum_{j=1}^{S-1} \gamma_j \Gamma_{j+1}^S \prod_{s=1}^{j-1} B_s + \Gamma_1^S > \gamma_S + \sum_{j=1}^{S-1} \gamma_j \Gamma_{j+1}^S + \Gamma_1^S = 1.$$

Therefore, $\theta_0^\circ < 1$.

[37] Note that

$$\gamma_S + \sum_{j=1}^{S-1} \frac{\gamma_j \Gamma_{j+1}^S}{\prod_{s=j}^{S-1} B_s} + \frac{\Gamma_1^S}{\prod_{s=1}^{S-1} B_s} < \gamma_S + \sum_{j=1}^{S-1} \gamma_j \Gamma_{j+1}^S + \Gamma_1^S = 1,$$

because $B_s > 1$ for every s.